

Name: Solutions

1. Use the method of elimination to reduce the following matrix to echelon form.

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 2 & 4 & 4 & 1 & 5 \\ -1 & -2 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 2 & 4 & 4 & 1 & 5 \\ -1 & -2 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 + R_1}} \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & -2 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

2. Determine all elements of the null space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 2 & 4 & 4 & 1 & 5 \\ -1 & -2 & -3 & 0 & 0 \end{bmatrix}$$

from the previous problem.

$$\begin{array}{ccccc} p & f & p & p & f \\ \left[\begin{array}{ccccc} 1 & 2 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right] \end{array}$$

$$x_2 = 1, x_5 = 0$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$x_5 = 0, x_3 = 0, x_1 = -2$$

by inspection

$$v_1 = (-2, 1, 0, 0, 0)$$

$$x_2 = 0, x_5 = 1$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = - \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

$$x_4 = -4$$

$$2x_3 - 4 = -3 \Rightarrow x_3 = 1/2$$

$$x_1 + \frac{1}{2} = -1 \Rightarrow x_1 = -3/2$$

$$v_2 = (-3/2, 0, 1/2, -4, 1)$$

3. One solution of $Ax = (1, 2, -1)$ is given by $(1, 0, 0, 0, 0)$. What are all the other solutions?

$$x + c_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3/2 \\ 0 \\ 1/2 \\ -4 \\ 1 \end{bmatrix}$$

c_1, c_2 arbitrary