Name: Solutions

1. Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 1 \end{bmatrix}.$$

a) **(2 points)** Explain briefly how you know, without doing any work, that this is a linearly dependent collection of vectors.

If k vectors in $\mathbb{R}^n$ are linearly independent, then $k \le n$.
We have 3 vectors in $\mathbb{R}^n$ $(k > n)$ so they are linearly dependent.

b) **(4 points)** Explicitly show that the collection is linearly dependent by writing v_3 as a linear combination of v_1 and v_2.

$$\left.\begin{aligned} \alpha_1 + 3\alpha_2 &= 7 \\ 3\alpha_1 - \alpha_2 &= 1 \end{aligned}\right] \text{ solve for } \alpha_1, \alpha_2$$

$\alpha_2 = 3\alpha_1 - 1$
$\Rightarrow 10\alpha_1 = 10$
$\Rightarrow \alpha_1 = 1$
$\Rightarrow \alpha_2 = 2$

Or by inspection:

$$v_1 + 2v_2 = v_3$$

c) **(2 points)** Find a set of numbers β_1, β_2 and β_3, not all zero, such that $\beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3 = 0$.

$$v_1 + 2v_2 - v_3 = 0$$

d) **(2 points)** One can compute that one solution of

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = \begin{bmatrix} 12 \\ 6 \end{bmatrix} \tag{1}$$

is $\alpha_1 = 2$, $\alpha_2 = 1$ and $\alpha_3 = 1$. Use your answer for part (c) to find a different linear combination of v_1, v_2 and v_3 that also equals $(12, 6)$. That is, find a different set of numbers α_1, α_2 and α_3 that also satisfies equation (1).

$\alpha_1' = \alpha_1 + \beta_1 = 2 + 1 = 3$
$\alpha_2' = \alpha_2 + \beta_2 = 1 + 2 = 3$
$\alpha_3' = \alpha_3 + \beta_3 = 1 + (-1) = 0$