

$$Q [q_1 \ q_2 \ q_3]$$

$$Q^T Q = \begin{bmatrix} q_1^T \\ q_2^T \\ q_3^T \end{bmatrix} [q_1 \ q_2 \ q_3]$$

85×3

$$A^T A = I$$

\Leftrightarrow cols of A are o.n.

$$= \begin{bmatrix} q_1^T q_1 & q_1^T q_2 & q_1^T q_3 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$A A^T = I \quad \Leftrightarrow \text{rows of } A \text{ are o.n.}$$

$$A = \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ p & f & p & f & p \\ \left[\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \end{array} \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}$$

x_1, x_3, x_5 pivot
 x_2, x_4 free

$$Ax = 0$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = 0$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} = -x_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} - x_4 \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = -x_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} - x_4 \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = -17 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 = 1 \quad x_4 = 0$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} * \\ 1 \\ * \\ 0 \\ * \end{bmatrix} \quad v_2 = \begin{bmatrix} * \\ 0 \\ * \\ 1 \\ * \end{bmatrix}$$

$$c_1 v_1 + c_2 v_2 = \begin{bmatrix} * \\ c_1 \\ * \\ c_2 \\ * \end{bmatrix}$$

$$c_1 v_1 + c_2 v_2 = 0$$

$$c_1 = 0, c_2 = 0$$

$\Rightarrow v_1, v_2$ are lin. indep

The Nullspace $N(A)$

is efficiently described by v_1, v_2

$$N(A) = \left\{ c_1 v_1 + c_2 v_2 : c_1, c_2 \in \mathbb{R} \right\}$$

No extra information

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$Ax = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left. \begin{array}{l} x_1 + 2x_2 + 3x_3 = 0 \\ 4x_1 + 5x_2 + 6x_3 = 0 \end{array} \right\}$$

solutions of

are same as

solutions of

$$\left. \begin{array}{l} x_1 + 2x_2 + 3x_3 = 0 \\ 0x_1 - 3x_2 - 6x_3 = 0 \end{array} \right\}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \xrightarrow{R_2 - 4R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 9 & 12 \end{bmatrix} \xrightarrow{\substack{R_2 - 4R_1 \\ R_3 - 6R_1}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -3 & -6 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 6 & -4 & 3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 6 & -4 & 3 \\ 3 & -1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \xrightarrow{R_2 - \frac{1}{2}R_1} \begin{bmatrix} 6 & -4 & 3 \\ 0 & 1 & \frac{1}{2} \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6 & -4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2} \begin{bmatrix} 6 & -4 & 3 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 9 & 12 \end{bmatrix} \xrightarrow{\substack{R_2 - 4R_1 \\ R_3 - 6R_1}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -3 & -6 \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 9 & 12 \end{bmatrix}$$

L
U
A

$$LU = A$$

$$A_x = b$$

1) $Lw = b$ forward subs

2) $Ux = w$ back subs