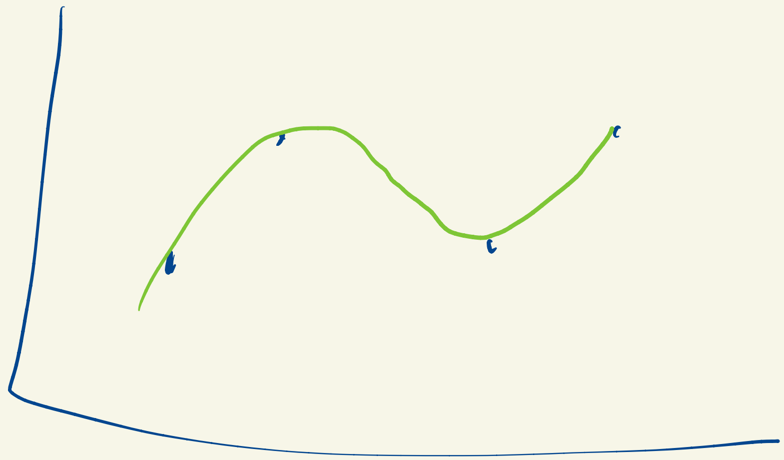
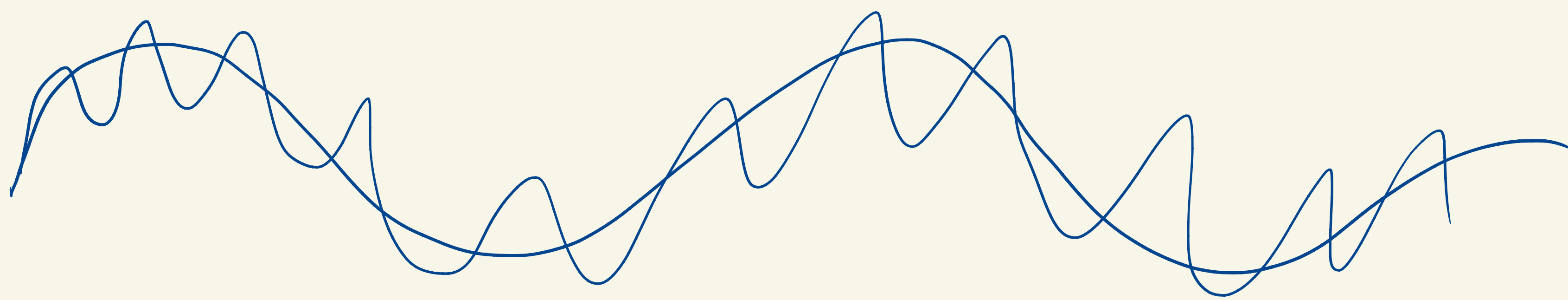


$a\ ,\ b$

$$a * b = b * a$$



$$x \longmapsto Ax$$

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

$$R^3 \to R^3$$

$$f(x, y, z) = x^2 + 15y + z$$

$$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$a \qquad\qquad b$$

$$f(a+b) = f(5,0,0) = 25$$

$$f(a) = 4 \qquad f(b) = 9$$

$$f(a) + f(b) = 13 \neq 25 = f(a+b)$$

Not linear.

$$x \longrightarrow \text{sorted } x$$

$$(3, 2, 1) \longmapsto (1, 2, 3)$$

$$(5, 1, 4) \longmapsto (1, 4, 5)$$

$$s(\overbrace{1, 0, 0}^{a}) = (0, 0, 1)$$

$$s(\underbrace{0, 0, 1}_{b}) = (0, 0, 1)$$

$$s(1, 0, 1) = (0, 1, 1)$$

$$\underbrace{\qquad\qquad}_{a+b}$$

$$s(a+b) = (0,1,1) \neq (0,0,2)$$

$$= (0,0,1) + (0,0,1)$$

$$= s(a) + s(b)$$

Not linear.

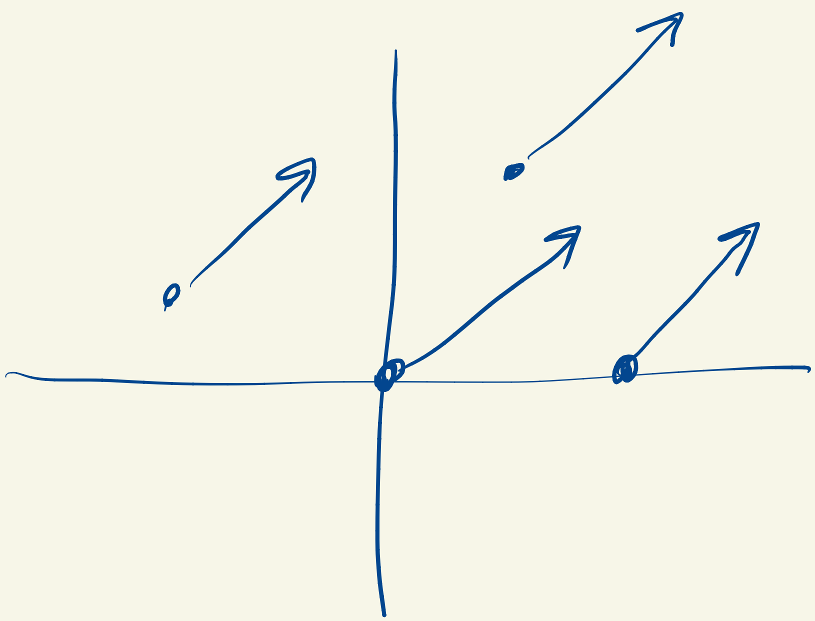
$f: \mathbb{R}^2 \to \mathbb{R}^2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

identity

$$f(a) = \overset{\frown}{a} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$a = \begin{bmatrix} x \\ y \end{bmatrix} \qquad f(x,y) = (x+1,\ y+1)$$



"translation"

g linear

$$g(0) = g(0+0) = g(0) + g(0)$$

$$g(0) = g(0) + g(0)$$

$$0 = g(0)$$

Every linear map takes 0 to 0

$\mathbb{R}^n$ $\mathbb{R}^m$

$$A0 = 0$$

Translation is an example of an affine map.

$f : \mathbb{R}^n \to \mathbb{R}^m$

We say $f(x)$ is affine if

$$f(x) = g(x) + b$$

where g is linear $\mathbb{R}^n \to \mathbb{R}^m$ and $b \in \mathbb{R}^m$

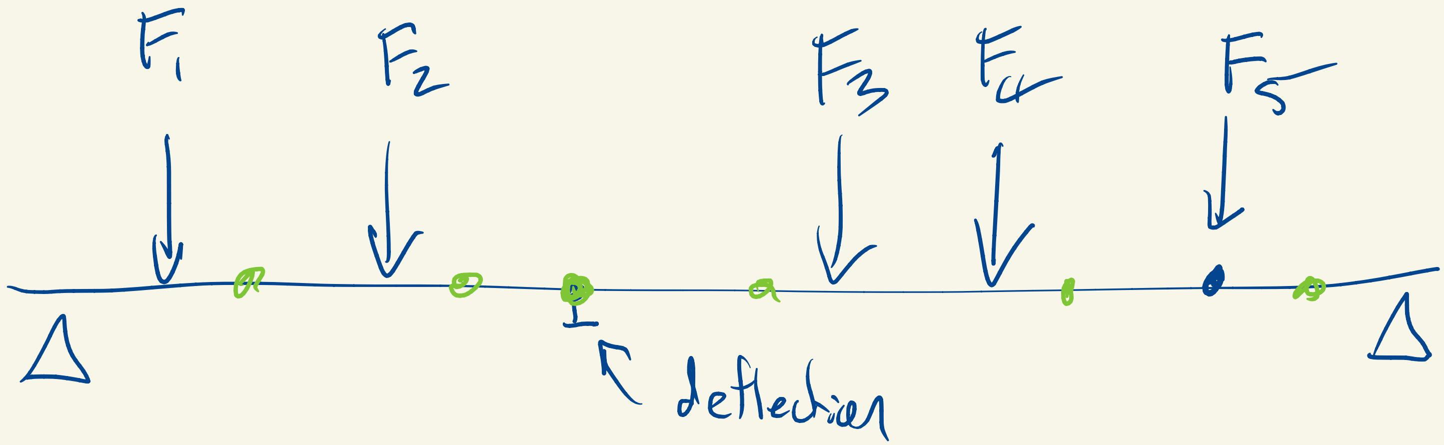
$$f(0) = \underbrace{g(0)} + b$$

$$= 0 + b = b$$

Affine functions satisfy limited superposition

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$$

$$\text{if} \quad \alpha + \beta = 1$$



$$
\begin{array}{cc}
 & n \text{ load points} \\
\begin{array}{c} m \\ \text{deflection} \\ \text{points} \end{array} & \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ c_{21} & & \vdots \\ \vdots & & \vdots \\ c_{m1} & & c_{mn} \end{bmatrix}
\end{array}
$$

compliance matrix C

c_{ij} has units of mm/kN

deflection per kN at load

c_{ij} is the amount of deflection at deflection point i induced by a unit load a load point j

3 load points

$$
\begin{array}{c}
\text{2 def. points}
\end{array}
\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}
=
\begin{bmatrix} c_{11}F_1 + c_{12}F_2 + c_{13}F_3 \\ c_{21}F_1 + c_{22}F_2 + c_{23}F_3 \end{bmatrix}
$$

Liner systems

$$3x + 2y = 9$$

$$-2x + 7y = 14$$

$$\left.\begin{aligned}
A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n &= b_1 \\
A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n &= b_2 \\
&\vdots \\
A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n &= b_m
\end{aligned}\right\} m \text{ eq's}$$

We know A_{ij}'s and the b_i's.

Want to determine the x_j's.

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & \cdots & \cdots & A_{2n} \\ \vdots & & & \\ A_{m1} & \cdots & \cdots & A_{mn} \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Given A and b find x so that

$$Ax = b$$

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & \cdots & \cdots & A_{2n} \\ \vdots & & & \\ A_{m1} & \cdots & \cdots & A_{mn} \end{bmatrix}$$

$$\begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix}$$

columns of A

$$Ax = \begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 a_1 + x_2 a_2 \cdots + x_n a_n$$

$$x_1 a_1 + x_2 a_2 \cdots + x_n a_n = x_1 \begin{bmatrix} A_{11} \\ A_{21} \\ \vdots \\ A_{m1} \end{bmatrix} + x_2 \begin{bmatrix} A_{12} \\ \vdots \\ A_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} A_{1n} \\ \vdots \\ A_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} x_1 + A_{12} x_2 + \cdots + A_{1n} x_n \\ \vdots \\ A_{m1} x_1 \cdots + A_{mn} x_n \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 8 \end{bmatrix}$$

back substitution

$$x_3 = 2$$

$$2x_2 - x_3 = -4$$

$$2x_2 - 2 = -4$$

$$x_2 = -1$$

$$x_1 = 3$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 8 \end{bmatrix}$$

no solutions!

$$0 \cdot x_1 + 0 \cdot x_1 + 0\, x_3 = 0$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$$

no solution

$$x_3 = 4$$

$$-x_3 = 1$$

$$x_3 = -1$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \checkmark$$ there is a solution!

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$ is another solution!