f

$$\hat{f}(x) = f(w) + (\nabla f)^T (x - w)$$

If we are approximating f at $w = 0$

and if $f(0) = 0$ this becomes

∇f

$$\hat{f}(x) = (\nabla f)^T x = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

<u>Say was an approximation</u> $\hat{s}$ vs true s

$$\hat{s}(m_1, \dots, m_3) = c^T m$$

The sensitivities are ∇s

$$10$$

$$\underbrace{0_{35}} \rightarrow \underbrace{(0, 0, \ldots, 0)}_{35}$$

$$x_{65}$$

$$1_{34}^{T} X_{65:99}$$

$$\begin{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \end{bmatrix} \qquad \begin{pmatrix} 0_{65} \\ 1_{35} \end{pmatrix}$$

x, y

(x, y)

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

$(x_1, \ldots, x_n, y_1, \ldots, y_m)$

$c = (0, 0, 1, 1)$

$x = (x_1, x_2, x_3, x_4)$

$$c^T \cdot x = 0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4$$

Linear Regression

Suppose we want to predict annual income of a person based on certain features

1) grad from HS y/n → 1 or 0
2) grad from university y/n → 1 or 0
3) has a post-grad degree/cert. y/n → 1 or 0
4) age over 20 years old → #

A model: $\hat{y} = b^T \cdot x + v$

hat is predicted (income)

contains features

parameters

$$x = (\underbrace{1, 1, 0}, \underbrace{17})$$

did grad from HS

did grad from uni

no post grad

37 year old

$$b = (\overbrace{b_1, b_2, b_3 \; b_4}^{\text{dollars}})$$

v is a number

$[\hat{y}] =$ dollars

$[v] =$ dollars

→ income of a 20 y/old who didn't grad from HS, etc.

b_1 is expected additional annual income for graduating HS

$[b_4] =$ dollars / year

additional income for getting older by one year regression: predicting a real number

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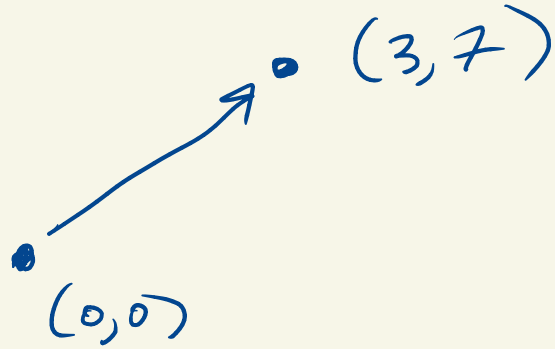
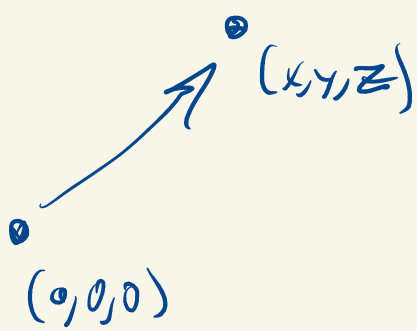
Chapter 3 Norms + Distance (+ Angles!)



How far is (3,7)

from (0,0)?

$$\sqrt{7^2+3^2} = \sqrt{58}$$



dist: $x^2+y^2+z^2$

$$\|x\| = \left(x_1^2 + x_2^2 + \cdots + x_n^2\right)^{1/2}$$

$\uparrow$

$\mathbb{R}^n$

The Euclidean norm of the vector x.

It is a measure of the size of x.

"distance from x to 0"

$x = (1, 2, 1, -4)$

$(-4)^2$

$$\|x\| = \left(1^2 + 2^2 + 1^2 + 4^2\right)^{1/2} = \sqrt{22}$$

Some properties of the norm $\sqrt{x_1^2 + \cdots + x_n^2}$

1. $\|x\| \geq 0$

2

$$\|x\| = 0 \iff x = 0$$

$$\underbrace{x_1^2}_{\geq 0} + \cdots + \underbrace{x_n^2}_{\geq 0} = 0$$

$$x_1^2 = 0 \Rightarrow x_1 = 0$$

$$\|7x\| = 7\|x\|$$

$$\left((7x_1)^2 + (7x_2)^2 + \cdots + (7x_n)^2\right)^{1/2}$$

$$= \left(7^2 x_1^2 + 7^2 x_2^2 + \cdots + 7^2 x_n^2\right)^{1/2}$$

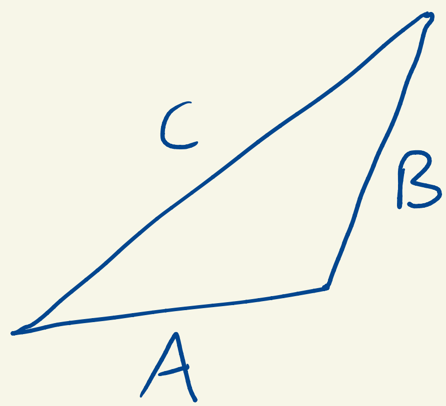
$$= \left(7^2 (x_1^2 + x_2^2 + \cdots + x_n^2)\right)^{1/2}$$

$$= (7^2)^{1/2} \|x\|$$

$$= 7\|x\|$$

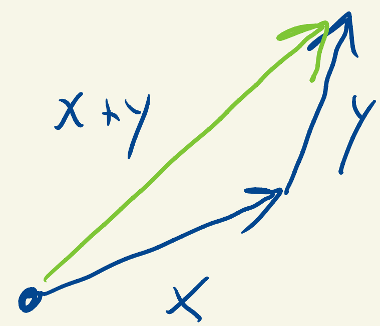
3 $\quad \|\alpha x\| = |\alpha| \, \|x\| \qquad \alpha \in \mathbb{R}$

$$(\alpha^2)^{1/2} = |\alpha|$$



$A + B \geq C$

4 $\quad \|x+y\| \leq \|x\| + \|y\|$



"Triangle inequality"

$$\left[(x_1+y_1)^2 + (x_2+y_2)^2 + \cdots + (x_n+y_n)^2\right]^{1/2}$$

A norm is a thing that satisfies properties 1 – 4.

The text defaults to the Euclidean norm.