

$(3, 2)$

(
↑
kg
of
flour
)
↑
liters
of
oil
)
↑
kg
of
sugar
)

Basic Operations

\mathbb{N}

$$a = (1, 3, 7, 5)$$

$a_1 \quad a_2 \quad a_3 \quad a_4$

indexing

$$a_3 = 7$$

subsetting

$$a_{2:3} = (3, 7)$$

$$a_{2:4} = (3, 7, 5)$$

concatenation

$$b = (2, -3, 9)$$

$$(a, b) = (1, 3, 7, 5, 2, -3, 9)$$

$$O_n = \underbrace{(0, 0, \dots, 0)}_n$$

0

zero vector

$$(0, 0, 1, 0, 0)$$

$$\underline{a+0}$$

$$(e_3)_5 = 0$$

$$\vec{1}_n = \underbrace{(1, 1, \dots, 1)}_n$$

$\vec{1}$ which vector $(e_j)_k$ entry
 $e_1, e_2, e_3, \dots, e_n$

standard basis vectors
(unit vectors)

$$e_k = \underbrace{(0, 0, \dots, 0, 1, 0, \dots, 0)}_n \text{ (suppressed)}$$

↑
slot k

Fundamental Vector Operations

1) Vector addition

2) Scalar multiplication

$$a = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$$

$$a + b = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix} \quad \begin{array}{l} 2 + 4 = 6 \\ 1 + 2 = 3 \\ 6 + (-2) = 4 \end{array}$$

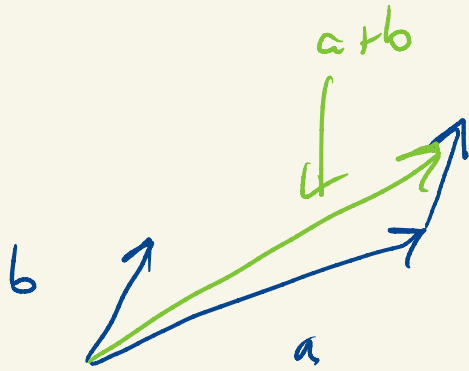
Portfolio assets (\quad , \quad , \quad)

of
shares of AAPL

mequn
portfolios

Time series audio signal

combining audio signals



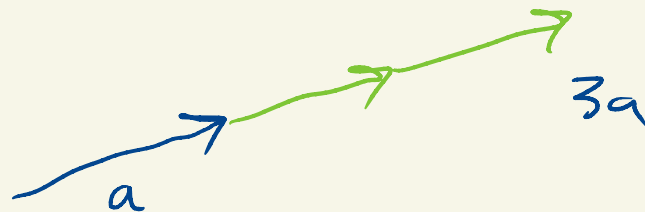
Scalar Multiplication

$$7 \cdot \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 28 \end{bmatrix}$$

audio signals

louder
or
quieter

port folio



These operations get along with each other

a, b, c vectors of same length

α, β numbers

$$a + b = b + a$$

$$(a + b) + c = a + (b + c)$$

$$(a + b + c)$$

$$a + 0 = a = 0 + a$$

$$a + (-a) = 0$$

$$-a = (-a_1, -a_2, \dots, -a_n)$$

↑

$$\alpha(\beta a) = (\alpha\beta)a$$

$$\alpha(a + b) = \alpha a + \alpha b$$

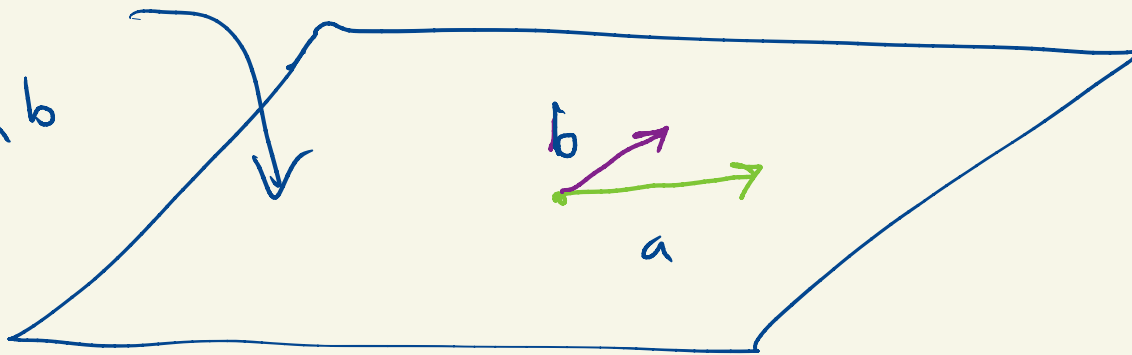
$$(\alpha + \beta)a = \alpha a + \beta a$$

a, b vectors α, β numbers

$$\alpha a + \beta b$$

→ a linear combination of a and b

all linear
combos
of a and b



$$a \cdot 1 + b \cdot 0$$

$$a \cdot 0 + b \cdot 0$$

Inner Product (duality) (dot product)

$$a = (a_1, a_2, a_3, a_4)$$

$$b = (b_1, b_2, b_3, b_4)$$

↪ transpose

$$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$

we are adding up the entries of b with weights coming from a .

b : portfolio assets

(# of shares, MSFT, AMZN)
of AAPL

a : price per share of
the same stocks

$$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

total value of portfolio

$$a = \vec{1}_4 \quad b = (b_1, b_2, b_3, b_4)$$

$$a^T b = b_1 + b_2 + b_3 + b_4$$

$$a = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \quad a^T b = \frac{1}{4} b_1 + \frac{1}{4} b_2 + \frac{1}{4} b_3 + \frac{1}{4} b_4$$
$$= \frac{1}{4} (b_1 + \dots + b_4)$$

$$a = e_3$$
$$= (0, 0, 1, 0)$$

$$a^T \cdot b = 0 \cdot b_1 + 0 \cdot b_2 + 1 \cdot b_3 + 0 \cdot b_4$$
$$= b_3$$

$$\begin{aligned} a^T \cdot b &= a_1 b_1 + a_2 b_2 + \dots + a_n b_n \\ &= b_1 a_1 + b_2 a_2 + \dots + b_n a_n \\ &= b^T a \end{aligned}$$