

1. Let

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix},$$

- a) Perform the Gram-Schmidt algorithm on these vectors (in this order) to determine orthonormal vectors  $q_1$ ,  $q_2$  and  $q_3$ .
- b) Write the vector  $d = (1, 1, 1, -3)$  as a linear combination of  $q_1$ ,  $q_2$  and  $q_3$ . Recall that because the  $q_i$ 's are orthonormal, the coefficients of the linear combination are given by  $q_i^T d$ .
- c) From part (b) you know that  $d$  is also a linear combination of the vectors  $a$ ,  $b$  and  $c$ . In fact, this is easy to spot. Using whatever technique you would like, write  $d$  as such a linear combination.
- d) If we performed Gram-Schmidt on the collection of vectors  $a$ ,  $b$ ,  $c$  and  $d$ , what would have happened? Be specific in your answer.