

1. Text 12.2 Modified. Suppose that the  $m \times n$  matrix  $Q$  has orthonormal columns.
  - a) Why do you immediately know that  $m \geq n$  and hence  $Q$  is tall or square?
  - b) Show that  $\hat{x} = Q^T b$  is the vector that minimizes  $J(x) = \|Qx - b\|^2$ . Do this by setting up the normal equations directly.
2. Supplemental 12.6 Hint: Your matrix  $A$  will be  $6 \times 4$  and will have a lot of 0's and 1's.
3. Suppose we want to compute the coefficients of the quadratic  $p(t) = c_1 + c_2 t + c_3 t^2$  that is the least squares fit to the following  $(t, y)$  data points:  $(0, 0)$ ,  $(1, 8)$ ,  $(3, 8)$ ,  $(4, 20)$ .
  1. Formulate the problem in the form of  $Ac = b$ . That is, find the matrix  $A$  and the vector  $b$ . You don't need to solve this system.
  2. Now formulate the normal equation  $A^t A c = A^t b$  that you would use to find these best-fit coefficients. Again, do not solve the system. (But don't worry, there is a Julia problem that tackles this!)
4. Text: problem 12.4. Hint: you will find that the square roots of the weights are important. For part (b), remember that the columns of  $C$  are linearly independent if and only if the only solution of  $Cx = 0$  is  $x = 0$ .