Name:

1. A piece of wire is parameterized by $\mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle$ for $0 \leq t \leq 2\pi$ and with density $\delta = z^2$. Compute the mass of the wire.

$$\vec{r}\,'(t) = \langle -\sin(t), \cos(t), 1 \rangle$$

$$\|\vec{r}\,'(t)\|^2 = \sin^2(t) + \cos^2(t) + 1 = 2$$

$$\int_C \delta \, ds = \int_0^{2\pi} \underbrace{t^2}_{z^2} \underbrace{\sqrt{2}}_{\|\vec{r}\,'\|} \, dt = \frac{\sqrt{2}t^3}{3} \Big|_0^{2\pi} = \boxed{\frac{8\sqrt{2}\pi^3}{3}}$$

2. The vector field $\mathbf{F} = \left\langle 3x^2y - y, 2y + x^3 - x \right\rangle$ is conservative. Find a potential for it.

$$f_x = 3x^2y - y \Rightarrow f = x^3y - xy + g(y)$$

$$\Rightarrow f_y = x^3 - x + g'(y)$$

But $f_y = 2y + x^3 - x \Rightarrow g'(y) = 2y$

$$\Rightarrow g(y) = y^2 + 7$$

$$\boxed{f(x,y) = x^3y - xy + y^2 + 7}$$

3. Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle xy, x \rangle$ and where C is the portion of the curve $y = x^2$ with $-1 \le x \le 1$.

$$\vec{r}(t) = \langle t, t^2 \rangle$$

$$\vec{r}\,'(t) = \langle 1, 2t \rangle$$

$$\vec{F}(\vec{r}(t)) = \langle t \cdot t^2, t \rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = t^3 + 2t^2$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{-1}^{1} t^3 + 2t^2 \, dt$$

$$= \frac{t^4}{4} + \frac{2t^3}{3} \Big|_{-1}^{1}$$

$$= \boxed{\frac{4}{3}}$$