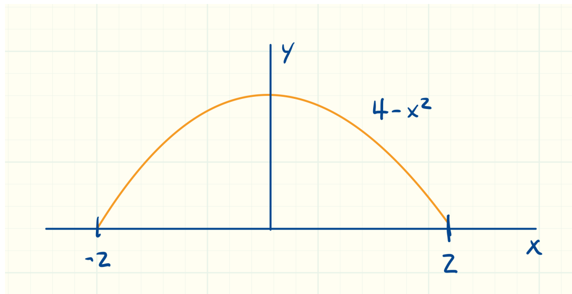
Name:

1. Let $\mathcal{E}$ be the 3-d region bounded determined by the inequalities $0 \le z \le 3x$ and $0 \le y \le 4 - x^2$. The figure below might help with visualizing the region.



a. Write down an iterated integral in terms of x, y and z variables that is equivalent to

$$\iiint_{\mathcal{E}} z\, dV.$$

Your innermost integral should be with respect to z, and the middle integral should be with respect to y. Do NOT compute the value of the integral.

$$\boxed{\int_{-2}^{2} \int_{0}^{4-x^2} \int_{0}^{3x} z \, dz \, dy \, dx}$$

b. For the integral you just wrote down, compute the two innermost integrals (i.e. with respect to z and then y) to reduce the triple integral to a single integral with respect to x. Do NOT further compute the value of the integral.

$$\int_0^{3x} z \, dz = \frac{z^2}{2} \Big|_0^{3x} = \frac{9x^2}{2}$$

$$\int_0^{4-x^2} \frac{9x^2}{2} \, dy = \frac{9x^2}{2} \cdot (4-x^2)$$

$$\boxed{\int_{-2}^{2} \frac{9}{2} x^2 (4-x^2) \, dx}$$

2. Rectangular coordinates (x, y, z) can be written in terms of spherical polar coordinates (ρ, θ, ϕ). Simply write down what these formulas are. I.e, $x =$ stuff involving ρ, θ and ϕ and so forth.

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

3. Let $\mathcal{E}$ be upper half sphere $\{(x, y, z) \mid x^2 + y^2 + z^2 \le 4\}$ of radius 2 with $z \ge 0$. Write the integral

$$\iiint_{\mathcal{E}} z^2 - x^2 - y^2 \, dV$$

in terms of spherical polar coordinates (ρ, θ, ϕ). Simplify the integrand to the extent possible, but do NOT compute the value of the integral.

$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^2 \left[\rho^2 \cos^2\phi - \rho^2 \sin^2\phi \left(\cos^2\theta + \sin^2\theta\right)\right] \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^2 \left(\cos^2\phi - \sin^2\phi\right) \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$= \boxed{\int_0^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^4 \left(\cos^2\phi - \sin^2\phi\right) \sin\phi \, d\rho \, d\theta \, d\phi}$$