MATH 253 Name: SOLUTIONS

Quiz 6 February 18, 2022

Instructions: Ten points total. Show all work for credit.

1. (5 pts.)

   (a) (4 pts.) Find (i) the best linear approximation to the function

$$f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \quad \text{at the point } (3, 2, 6)$$

and (ii) use it to approximate the value of $\sqrt{(3.01)^2 + (1.98)^2 + (6.01)^2}$, using a calculator to give your answer to four decimal places.

(i) Translation: "best linear approximation" $\equiv$ tangent plane equation

$$w - f(3,2,6) = f_x(3,2,6)(x-3) + f_y(3,2,6)(y-2) + f_z(3,2,6)(z-6)$$

Computations: $f(3,2,6) = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{49} = 7$ //

$f_x(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+y^2+z^2}}$; $f_x(3,2,6) = \frac{3}{7}$ //

By symmetry, $f_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}}$; $f_y(x,y,z) = \frac{2}{7}$ //

$f_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}$; $f_z(3,2,6) = \frac{6}{7}$ //

Tangent plane equation: $w - 7 = \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$

Simplifying: $\boxed{w = \frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z}$

(ii) $f(3.01, 1.98, 6.01) \approx$ plug into to tangent plane equation

Answers: (i) Linear approximation:

$z = \frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z$

↑

Equation of tangent plane

(ii) $\sqrt{(3.01)^2 + (1.98)^2 + (6.01)^2} \approx$

$f(3.01, 1.98, 6.01) \approx$

$\frac{3}{7}(3.01) + \frac{2}{7}(1.98) + \frac{6}{7}(6.01) \approx 7.0071$

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(b) (1 pt.) **(Visualization practice)** Describe carefully in words the level surface

$$f(x, y, z) = 12$$

$\sqrt{x^2+y^2+z^2} = 12 \Rightarrow x^2+y^2+z^2 = (12)^2$

Sphere of radius 12, centered at the origin

2. (a) (5 pts)

i. (2 pts.) Use the Chain Rule to find $\frac{\partial z}{\partial t}$ when $z = \arctan\left(x^2+y^2\right)$, for $x = s\ln(t)$ and $y = te^s$. (You **must** use the chain rule to earn credit for this part. No chain rule, no credit.) You may give your answer in terms of x, y, s, and t.

$z < \begin{matrix} x < \begin{matrix} s \\ t \end{matrix} \\ y < \begin{matrix} s \\ t \end{matrix} \end{matrix}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= \left(\frac{2x}{1+(x^2+y^2)^2}\right)\left[\frac{s}{t}\right] + \left(\frac{2y}{1+(x^2+y^2)^2}\right)e^s$$

$$= \boxed{\frac{2xs}{t(1+(x^2+y^2)^2)} + \frac{2ye^s}{1+(x^2+y^2)^2}}$$ (or equivalent)

$$\left(\frac{2xs}{t} + 2ye^s\right)\left(\frac{1}{1+(x^2+y^2)^2}\right)$$ etc.

ii. (2 pts.) Now compute the value $\left.\frac{\partial z}{\partial t}\right|_{(0,1)}$.

That is, compute the value of the partial derivative at the point $(s, t) = (0, 1)$.

$x = 0$ $\quad y = 1$
$s = 0$ $\quad t = 1$

At $s = 0$, $t = 1$, $x = 0\ln(1) = 0$, $y = 1e^0 = 1$. Thus

$$\left.\frac{\partial z}{\partial t}\right|_{(0,1)} = \frac{2(0)(0)}{1(1+(1)^2)} + \frac{2(1)e^0}{1+(1)^2} = 0 + \frac{2}{2} = \boxed{1}$$

iii. (1 pt.) Is the function z increasing, decreasing, or stable in the t-direction at the point $(s, t) = (0, 1)$? Justify briefly.

Since $1 > 0$, z is $\boxed{\text{increasing}}$ in the t-direction at $(0,1)$.