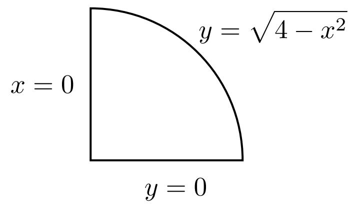
Name:

1. Let $\mathcal{E}$ be the 3-d region determined by the inequalities $y \geq 0$, $x \geq 0$, $x^2 + y^2 \leq 4$ and $0 \leq z \leq y$. The following region in the x-y plane might help you visualize some of these inequalites.



a. Write down an iterated integral in terms of x, y and z variables that is equivalent to

$$\iiint_{\mathcal{E}} 2x \, dV.$$

Do NOT compute the value of the integral.

$$\boxed{\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^y 2x \, dz \, dy \, dx} = I$$

b. Ok, now go ahead and compute the value of the integral.

$$I = \int_0^2 \int_0^{\sqrt{4-x^2}} 2xy \, dy \, dx$$

$$= \int_0^2 xy^2 \Big|_0^{\sqrt{4-x^2}} dx = \int_0^2 x(4-x^2) \, dx$$

$$= \int_0^2 4x - x^3 \, dx$$

$$= \frac{4x^2}{2} - \frac{x^4}{4} \Big|_0^2 = \frac{16}{2} - \frac{16}{4} = \boxed{4}$$

2. Rectangular coordinates (x, y, z) can be written in terms of spherical polar coordinates (ρ, θ, ϕ). Simply write down what these formulas are. I.e, x = stuff involving ρ, θ and ϕ and so forth.

$$z = \rho\cos\phi$$

$$x = \rho\sin\phi\cos\theta$$

$$y = \rho\sin\phi\sin\theta$$

3. Let $\mathcal{E}$ be the sphere $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 9\}$ of radius 3. Write the integral

$$\iiint_{\mathcal{E}} x^2 + y^2 \, dV$$

in terms of spherical polar coordinates (ρ, θ, ϕ). Simplify the integrand to the extent possible, but do NOT compute the value of the integral.

$$x^2+y^2 = (\rho\sin\phi\cos\theta)^2 + (\rho\sin\phi\sin\theta)^2$$

$$= \rho^2\sin^2\phi\left(\cos^2\theta + \sin^2\theta\right)$$

$$= \rho^2\sin^2\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{3} \rho^2\sin^2\phi \; \rho^2\sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{3} \rho^4\sin^3\phi \, d\rho \, d\phi \, d\theta$$