Name: Solutions

1. A quantity of gas is housed in an adjustable container. Its pressure P in kPa, temperature T in Kelvin, and volume V in liters satisfy

$$PV = 8T.$$

Suppose at time $t = 0$(measured in seconds) that the system that the volume and the temperature of the gas are changing and:

$$V = 20\ell \quad (1)$$
$$T = 300K \quad (2)$$
$$\frac{dV}{dt} = 0.1\frac{\ell}{s} \quad (3)$$
$$\frac{dT}{dt} = 0.2\frac{K}{s}. \quad (4)$$

1. What is the pressure of the gas at time $t = 0$?

$$P = \frac{8T}{V} = \frac{8 \cdot 300}{20} = \boxed{120 \text{ kPa}}$$

2. Use the chain rule to compute dP/dt at time $t = 0$.

$$\frac{dP}{dt} = \frac{\partial P}{\partial V} \cdot \frac{dV}{dt} + \frac{\partial P}{\partial T} \cdot \frac{dT}{dt}$$

$$= -\frac{8T}{V^2} \cdot \frac{dV}{dt} + \frac{8}{V} \cdot \frac{dT}{dt}$$

$$= -\frac{8 \cdot 300}{(20)^2} \cdot \frac{1}{10} + \frac{8}{20} \cdot \frac{2}{10}$$

$$= -\frac{6}{10} + \frac{8}{100} = -\frac{52}{100} = -\frac{26}{50} = -\frac{13}{25}$$

2. Suppose a temperature field $T(x, y)$ satisfies $\nabla T = \langle y - 4, x + 2y \rangle$. A bug follows a path $\mathbf{r}(t) = \left\langle -t, t^2 \right\rangle$. At what times t does the bug report that $d/dt\, T(\mathbf{r}(t)) = 0$?

We want $\vec{\nabla} T \cdot \vec{r}\,' = 0$.

$$\vec{r}\,' = \langle -1, 2t \rangle$$

At time t, $\vec{\nabla} T = \langle y-4, x+2y \rangle \Big|_{x=-t,\, y=t^2}$

$$= \langle t^2-4, -t+2t^2 \rangle$$

$$\vec{\nabla} T \cdot \vec{r}\,' = \langle t^2-4, -t+2t^2 \rangle \cdot \langle -1, 2t \rangle$$

$$= -t^2 + 4 - 2t^2 + 4t^3$$

$$= 4t^3 - 3t^2 + 4$$

So $4t^3 - 3t^2 + 4 = 0$.

(My bad; misprint led to cubic; full credit for this.)

Numerically, just one root at

$$t \approx -0.8$$