Name: Solutions

1. The following vector field is conservative:

$$\mathbf{F} = \left\langle y\cos(xy), x\cos(xy) + 3y^2 \right\rangle$$

a) Find **all** potential functions for **F**.

$$\frac{\partial f}{\partial x} = y\cos(xy) \Rightarrow f(x,y) = \sin(xy) + g(y)$$

$$\left.\begin{aligned} \frac{\partial f}{\partial y} &= x\cos(xy) + g'(y) \\ \frac{\partial f}{\partial y} &= x\cos(xy) + 3y^2 \end{aligned}\right] \Rightarrow g(y) = y^3 + C$$

$$\boxed{f(x,y) = \sin(xy) + y^3 + C}$$

b) Doing very little work, compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the straight line from the origin to the point $(1, \pi)$.

$$f(1,\pi) - f(0,0) = \left(\sin(\pi) + \pi^3\right) - \left(\sin(0) + 0^3\right)$$

$$\boxed{= \pi^3}$$

2. Recall that Green's Theorem states that for any curve C traversing the boundary (counterclockwise) of a simply connected region $\mathcal{D}$

$$\int_C P\,dx + Q\,dy = \iint_{\mathcal{D}} \left(-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\right) dA.$$

Use Green's theorem to compute the line integral $\int_C y^3\,dx - x^3\,dy$ where C is the circle $x^2 + y^2 = 9$ given the counter clockwise orientation. For full credit, your solution must employ Green's Theorem.

$$\int_C y^3\,dx - x^3\,dy = \iint_{\mathcal{D}} (-3y^2 - 3x^2)\,dA(x,y)$$

$$= \int_0^{2\pi}\int_0^3 -3r^2\,r\,dr\,d\theta$$

$$= \int_0^{2\pi} -\frac{3}{4}r^4\Big|_0^3\,d\theta$$

$$= -\frac{2\pi}{4}3^5$$

$$\boxed{= -\frac{243\pi}{2}}$$