Quiz 2
Math 253

Name: Key

09/04/19

1. Determine the vector projection of the vector $\mathbf{a} = \langle 2, 1, -3\rangle$ onto the vector $\mathbf{b} = \langle -3, 2, 2\rangle$.

$$\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b} = \frac{-6+2-6}{9+4+4}\langle -3,2,2\rangle$$

$$= \frac{-10}{17}\langle -3,2,2\rangle = \langle \frac{30}{17}, \frac{-20}{17}, \frac{-20}{17}\rangle$$

The scalar projection is $\frac{a\cdot b}{\|b\|} = \frac{-10}{\sqrt{17}}$

2. Determine the angle between the vectors $\mathbf{x} = \langle 1, 1, 4\rangle$ and $\mathbf{y} = \langle -2, 1, 1\rangle$. You may leave your answer in a form involving an inverse trigonometric function.

$$\vec{x}\cdot\vec{y} = \|\vec{x}\|\,\|\vec{y}\|\cos\theta$$

$$-2+1+4 = \sqrt{1+1+16}\sqrt{4+1+1}\cos\theta$$

$$3 = \sqrt{18}\sqrt{6}\cos\theta$$

$$3 = 6\sqrt{3}\cos\theta$$

$$\frac{1}{2\sqrt{3}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

3. Let $\mathbf{v} = \langle 2, 0, 1\rangle$ and $\mathbf{w} = \langle 1, -2, 3\rangle$. Compute $\mathbf{v}\times\mathbf{w}$.

$$\vec{v}\times\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(6-1) + \hat{k}(-4-0)$$

$$= \langle 2, -5, -4\rangle$$