Exam 2 Name:____________________

Math 202 F01 11/12/12

1. (12 pts.) A 3-d object is bounded below by $z = \sqrt{x^2 + y^2}$ and above by $z = 6 - x^2 - y^2$. Its mass density is given by $\rho(x, y, z) = x^2 + y^2 + z^2$. Using CYLINDRICAL COORDINATES, set up an appropriate integral expression for $\bar{z}$, the z-coordinate of its center of mass. DO NOT evaluate the integrals.

2. (12 pts.) Consider the function $f(x, y) = x^3 + y^3 - 3xy + 12$.

   (a) (4 pts.) Show that $(0, 0)$ and $(1, 1)$ are critical points of f. (They are actually the only critical points of f, but you need not show that.)

   (b) (8 pts.) Apply the 2nd derivative test at each of these points, and state your conclusions from it.

3. (7 pts.) Give an equation for the tangent plane to the surface $x^2 - 2y^2 + z^2 + yz = 2$ at the point $(2, 1, -1)$.

4. (15 pts.–5 pts. each) The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2},$$

where T is measured in $^\circ C$, and x, y, z are measured in meters.

(a) Find the rate of change of the temperature at the point $(2, -1, 2)$ in the direction towards the point $(3, -3, 3)$. GIVE UNITS.

(b) At $(2, -1, 2)$, in what direction does T increase most rapidly?

(c) What is the maximum rate of change of T at $(2, -1, 2)$, among all directions?

5. (12 pts.) Reverse the order of integration in the following integral, and then evaluate it.

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\, dx\, dy$$

6. (10 pts.) Ohm's law states that in an electrical circuit the current, I, depends on the voltage, V, and resistance, R, by

$$I = V/R.$$

Suppose at some moment $R = 100$ ohms, $V = 32$ volts, $dR/dt = 0.03$ ohms/s, and $dV/dt = -0.01$ volts/s. Determine dI/dt at that moment. GIVE UNITS. (Hint: Use the multivariable chain rule. The unit 'volt/ohm' is also called an 'ampere'.)

7. (12 pts.) Use the method of Lagrange multipliers to find the point on the sphere $x^2 + y^2 + z^2 = 70$ that minimizes $f(x, y, z) = 2x + 6y + 10z$.

8. (12 pts.–3 pts. each) Complete the following.

(a) The average value of a function $f(x, y)$ over a 2-dimensional region R is given by the formula:

(b) In spherical coordinates, dV is:

(c) $\lim_{(x,y)\to(0,0)} \dfrac{2x^2 - xy^2}{x^2 + y^2}$ does not exist since:

(d) The geometric relationship between the level curves of a function $z = f(x, y)$ and the gradient vectors $\nabla f(x, y)$ is:

9. (8 pts.–4 pts. each) Suppose $x = u^2 + v$, $y = u - v^2$ represents a change of coordinates for re-expressing a double integral in x, y in terms of u, v.

(a) Compute $\dfrac{\partial(x, y)}{\partial(u, v)}$.

(b) Give a sentence or two of informal explanation of why $\left|\dfrac{\partial(x, y)}{\partial(u, v)}\right| du\, dv$ should appear in the integral in terms of u, v. What do the parts of this expression represent geometrically? (DO NOT give a mathematical derivation of the expression.)