MATH202X-F01/UX1
Spring 2015

Practice Final Exam

Name: ______________________

Instructions. (100 points) You have 120 minutes. Each problem is worth 10 points. No calculators allowed. *Show all your work* in order to receive full credit.

(10^{pts}) **1.** Consider the following three points: $A(-1, 0, 1)$, $B(1, 1, 2)$, and $C(1, 2, 0)$.

(a) (5 pts) Determine whether the three points are collinear.

(b) (5 pts) If they are collinear, give the parametric equations of the line they form. If not, give the equation of the plane containing these three points.

(10$^{\text{pts}}$) **2.** Consider the plane:

$$x + 2y + 3z + 4 = 0$$

and the following symmetric equations for two distinct lines:

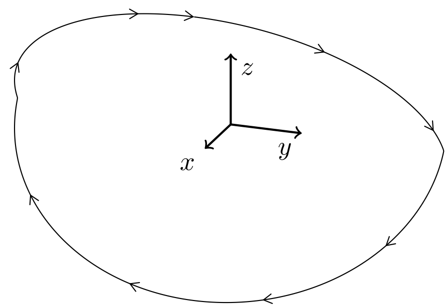
Line 1: $$\frac{x+1}{2} = y = \frac{z+1}{-1},$$

Line 2: $$x - 1 = y - 2 = \frac{z}{-1}.$$

Classify the intersection of the plane with each of the lines. Is there a one-point intersection (if so, give the coordinates of the point), no intersection because the line is parallel to the plane, or is the line in the plane?

(10pts) **3.** Consider the following vector-valued function, representing the trajectory of a particle:

$$\vec{r}(t) = \sqrt{1+\cos 2t}\,\vec{\imath} + 3\sin t\,\vec{\jmath} + 2\cos t\,\vec{k}.$$



(a) (5 pts) Find *all* the open interval(s) on which $\vec{r}(t)$ is smooth.

(b) (5 pts) Find the speed of the particle at $t = 0$.

Now, for extra credit: the parametric curve lies entirely on which of the following surface(s)? Check all that apply. You need not justify your answers.

☐ the ellipsoid: $x^2 + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$,

☐ the hyperboloid of one sheet: $x^2 + \dfrac{y^2}{9} - \dfrac{z^2}{4} = 1$,

☐ the elliptic cylinder: $\dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$,

☐ the hyperbolic paraboloid: $x = \dfrac{y^2}{9} - \dfrac{z^2}{4}$.

(10pts) **4.** A particle in space moves with acceleration:

$$\vec{a}(t) = \left\langle 1, \frac{1}{2\sqrt{t}}, 0 \right\rangle \qquad , \qquad t \geq 1$$

such that its velocity at $t = 1$ is $\vec{v}(1) = \left\langle \frac{3}{2}, 1, \frac{\sqrt{3}}{2} \right\rangle$ and its position is $\vec{r}(1) = \left\langle 1, \frac{2}{3}, \sqrt{3} \right\rangle$.

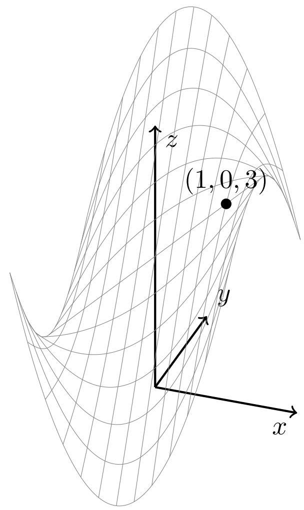
(a) (5 pts) Find the position of the particle at $t = 4$.

(b) (5 pts) Find the length of the curve between $t = 1$ and $t = 4$.

(10pts) **5.** Consider a point $P(1, 0)$ in the domain of the surface

$$z = x \cos y - yx^2 + 2(y + 1).$$

Assume the surface represents a hilly area, modeled below:



(a) (5 pts) What is the rate of change of altitude at the point P when moving in the direction of the vector $\vec{v} = \langle 3, 4 \rangle$?

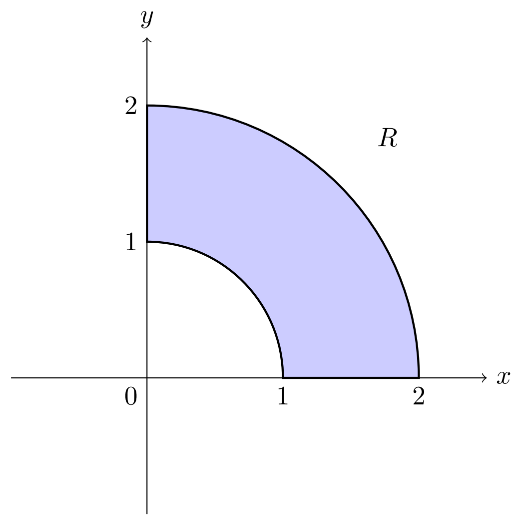
(b) (5 pts) What is the direction of maximum decrease at P? I.e. if chased by a bear, which direction should P take to get down that hill the fastest? What is the rate of decrease in that direction?

(10pts) **6.** Classify any critical points and then use Lagrange multipliers on the boundary and find the absolute maximum and minimum values of the function

$$f(x, y) = 2x^2 + 3y^2 - 4x - 5$$

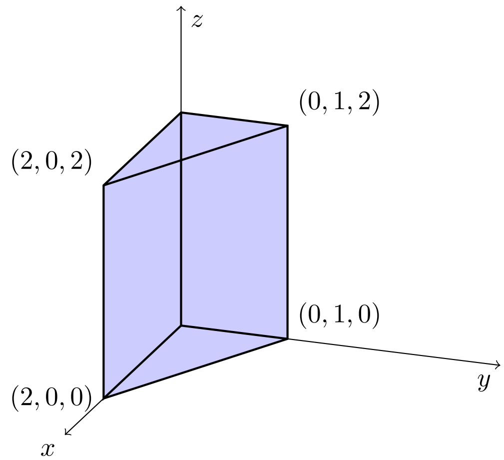
on the domain $x^2 + y^2 \leq 16$, $y \geq 0$.

(10pts) **7.** Find the moment of inertia about the $y-$axis I_y for a planar lamina R corresponding to the region below.



where the density $\rho(x, y) = y$.

(10pts) **8.** Consider the vector field $\overrightarrow{F}(x,y,z) = \left\langle x^2y, xy^2, 2xyz \right\rangle$ acting on a closed surface S consisting of the boundary of a triangular prism with the following vertices:



We are interested in evaluating the flux of the vector field over the surface S. Since the component functions of $\overrightarrow{F}$ have continuous first partial derivatives over the solid prism Q , apply the Divergence Theorem

$$\underbrace{\oiint_S \overrightarrow{F} \cdot \overrightarrow{N}\, dS}_{\text{flux}} = \iiint_Q \operatorname{div} \overrightarrow{F}\, dV$$

to evaluate the flux indirectly.

For extra credit, evaluate directly the flux. Note that you need to consider 5 surfaces separately including one which can be given by the following parametric representation: $\vec{r}(u,v) = \langle 2-2u, u, v\rangle$ for $0 \le u \le 1,\ 0 \le v \le 2$.

(10pts) **9.** Consider a particle moving through space along the curve C given by the following parametric representation:

$$\vec{r}(t) = \left\langle t^3 - 3t + 1, \frac{t}{2} + 1, \frac{t}{2}\cos \pi t \right\rangle \qquad , \qquad 0 \le t \le 2$$

and subject to the vector field: $\overrightarrow{F}(x, y, z) = \langle y^3 - 2xz, 3xy^2 + 2z, 2y - x^2 \rangle$.

(a) (3 pts) Show that $\overrightarrow{F}$ is conservative.

(b) (4 pts) Find *all* potential functions for the field $\overrightarrow{F}$.

(c) (3 pts) Use the Fundamental Theorem of Line Integrals to compute the work done on the particle.

For extra credit, using the same initial and final points, find a simpler path between them and use it to compute the work again – directly this time.

(10$^{\text{pts}}$) **10.** Use Green's Theorem to evaluate

$$\oint_C \big(\sin(x^2) + 3y\big)\,dx + (\ln y + 4x)\,dy$$

where C is the closed curve composed of the graph of $y = \frac{x^2}{4} + 1$ for $0 \le x \le 2$ followed by the line segment going from $(2,2)$ to $(0,1)$ as illustrated below: