$s(t) = \|\vec{r}'(t)\| = \text{speed}$

$$\vec{r}'(t) = s(t)\,T(t)$$

$$\vec{r}''(t) = s'(t)\,T + s\,T'(t)$$

$$= s'(t)\,T + s\,\|T'\|\,\frac{T'}{\|T'\|}$$

$$= s'(t)\,\vec{T} + s\,\|T'\|\,\vec{N}$$

Acceleration has two components one tangental and the other Normal.

Tangental component: $s'(t)$ how is the speed changes.

Normal is about turning instead.

$$\vec{r}''(t) \cdot \vec{T} = \underbrace{s'(t)}_{a_T}$$

$$\vec{r}''(t) \cdot \vec{N} = a_N$$ normal component of acceleration.

(usually, $\| \vec{r}'' - \vec{r}'' \cdot \vec{T} \|$) cuz $\vec{N}$ is a pain!

$$\vec{r}(t) = \langle \cos(t^2), \sin(t^2) \rangle$$

$$\vec{r}'(t) = \langle -2t\sin(t^2), 2t\cos(t^2) \rangle$$

$$\vec{T}(t) = \langle -\sin(t^2), \cos(t^2) \rangle$$

$$\vec{r}''(t) = \langle -2\sin(t^2), 2\cos(t^2) \rangle + \langle -4t^2\cos(t^2), -4t^2\sin(t^2) \rangle$$

$$\vec{T} \cdot \vec{r}'' = +2$$

tangential component $a_T = 2$

$$\vec{r}''(t) - \vec{T} \cdot \vec{r}'' \, \vec{T} = -4t^2 \langle \cos(t^2), \sin(t^2) \rangle$$

$$\| \vec{r}''(t) - \vec{T} \cdot \vec{r}'' \, \vec{T} \| = \underbrace{4t^2}_{a_N}$$

$$\vec{T}' = \langle -2t\cos(t^2), 2t\sin(t^2) \rangle$$

$$\vec{N} = \langle \cos(t^2), \sin(t^2) \rangle$$

Section 14.1 Multivariate functions

$$PV = nRT$$

# of particles

gas constant

temp (K)

volume

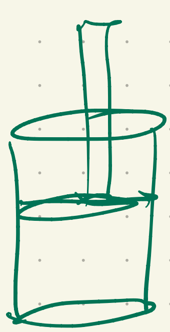
pressure.

Let us suppose n is fixed but V, T are not.

$P = (nR)\, T/V$ determines pressure

as a function of temp

and volume

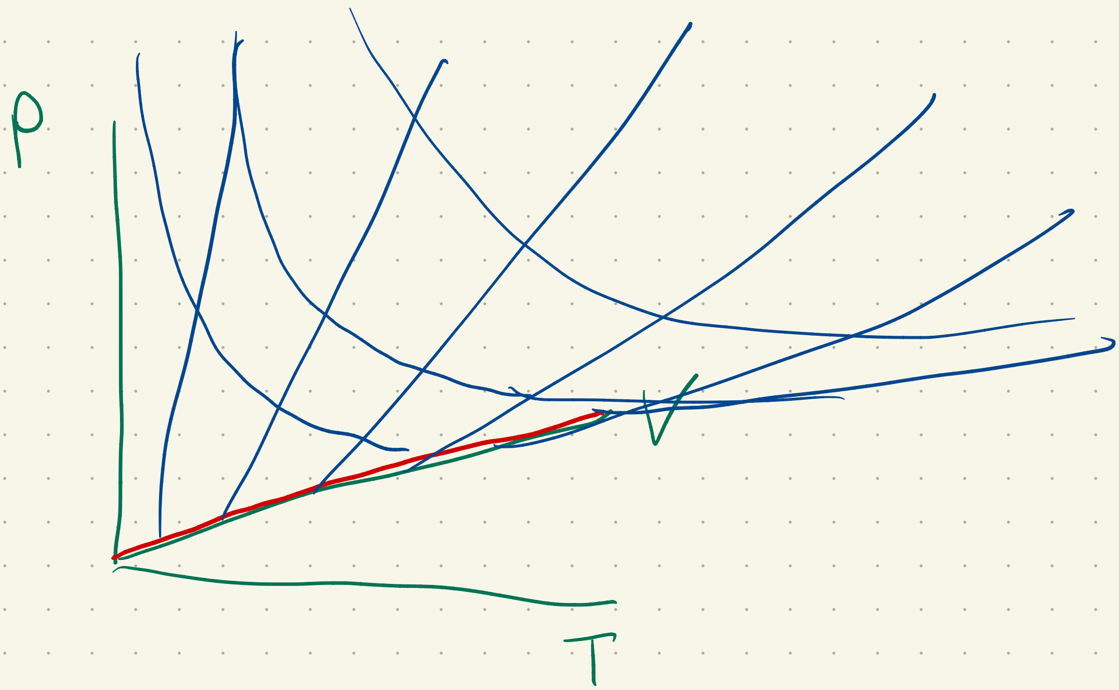


you controll T and V.

P is told back to you

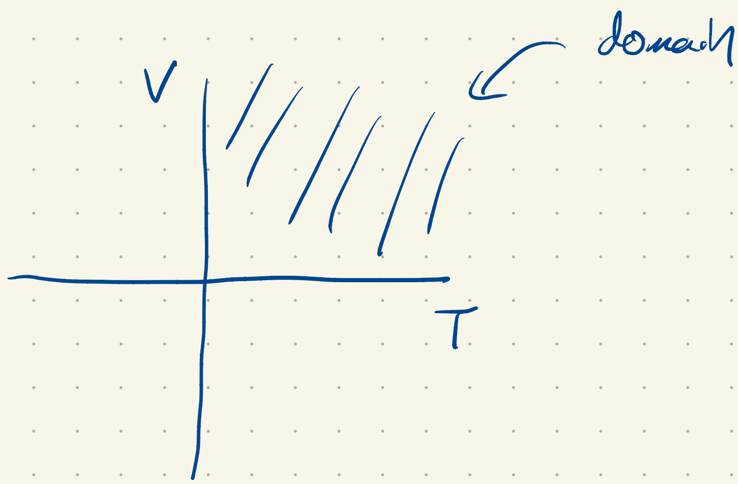
V goes up, P goes down

T goes up, P goes up



Vocab: $f(x,y)$ domain: allowable input

range all outputs



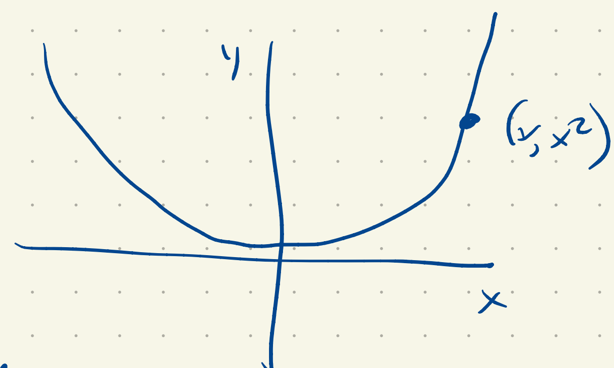
In above: $T > 0$

$V > 0$.

Range: $P > 0$

Let's visualize some functions of x, y.

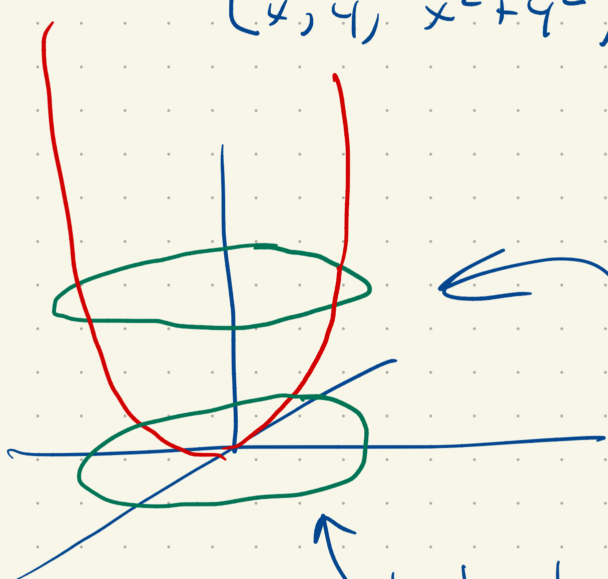
$f(x,y) = x^2 + y^2$



Graph: $(x, y, z = f(x,y))$

$(x, y = f(x))$ in old days

(x, y, x^2+y^2)



$z = x^2$ if $y = 0$

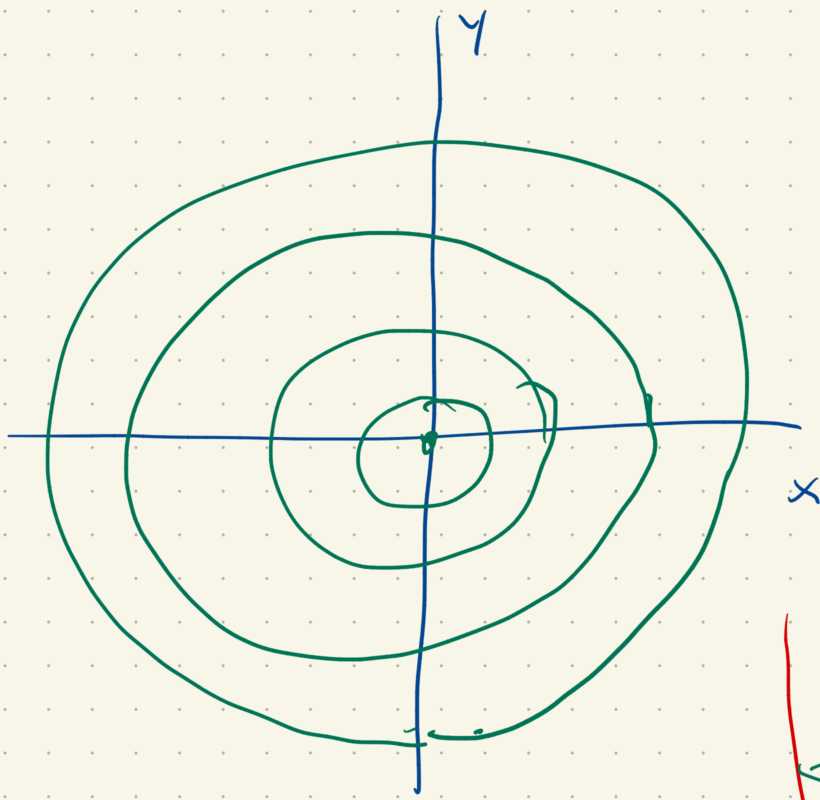
$z = y^2$ if $x = 0$

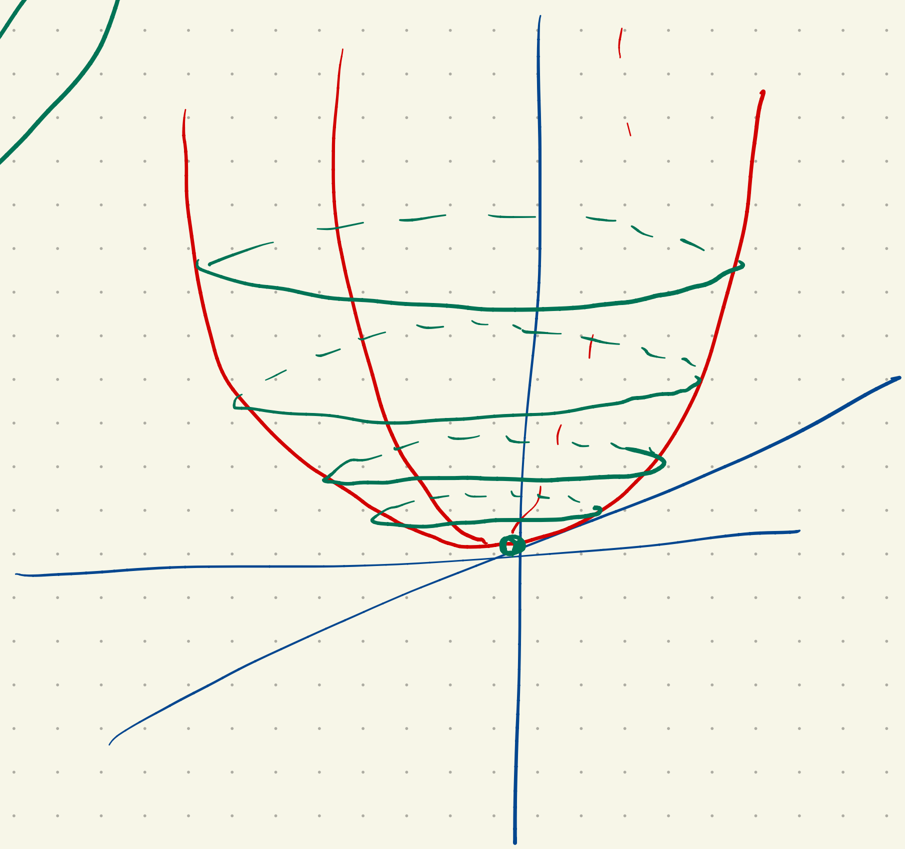
$\{ (x,y) : x^2+y^2 = c \}$

circle

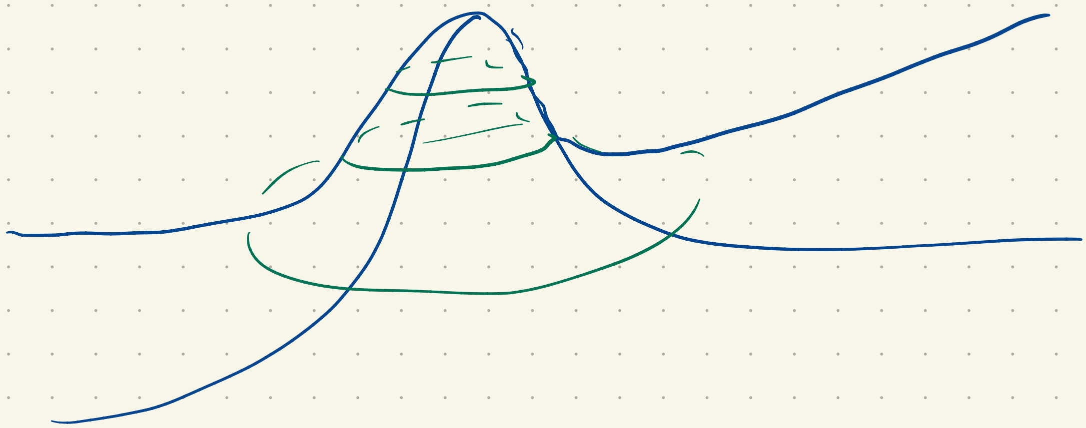
level set

Contour plot

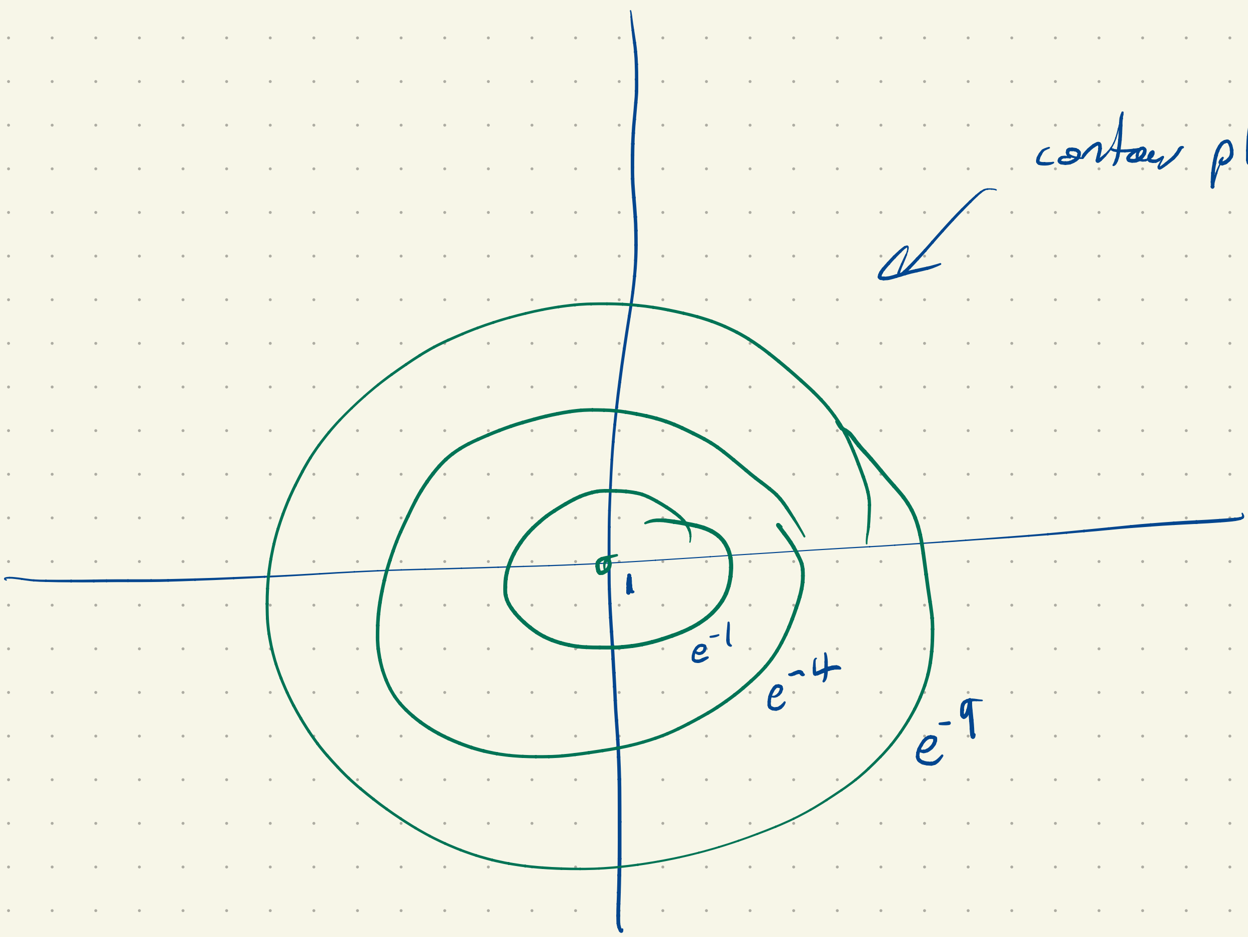




e.g. $\qquad f(x,y) = \exp(-x^2 - y^2)$



contour plot.

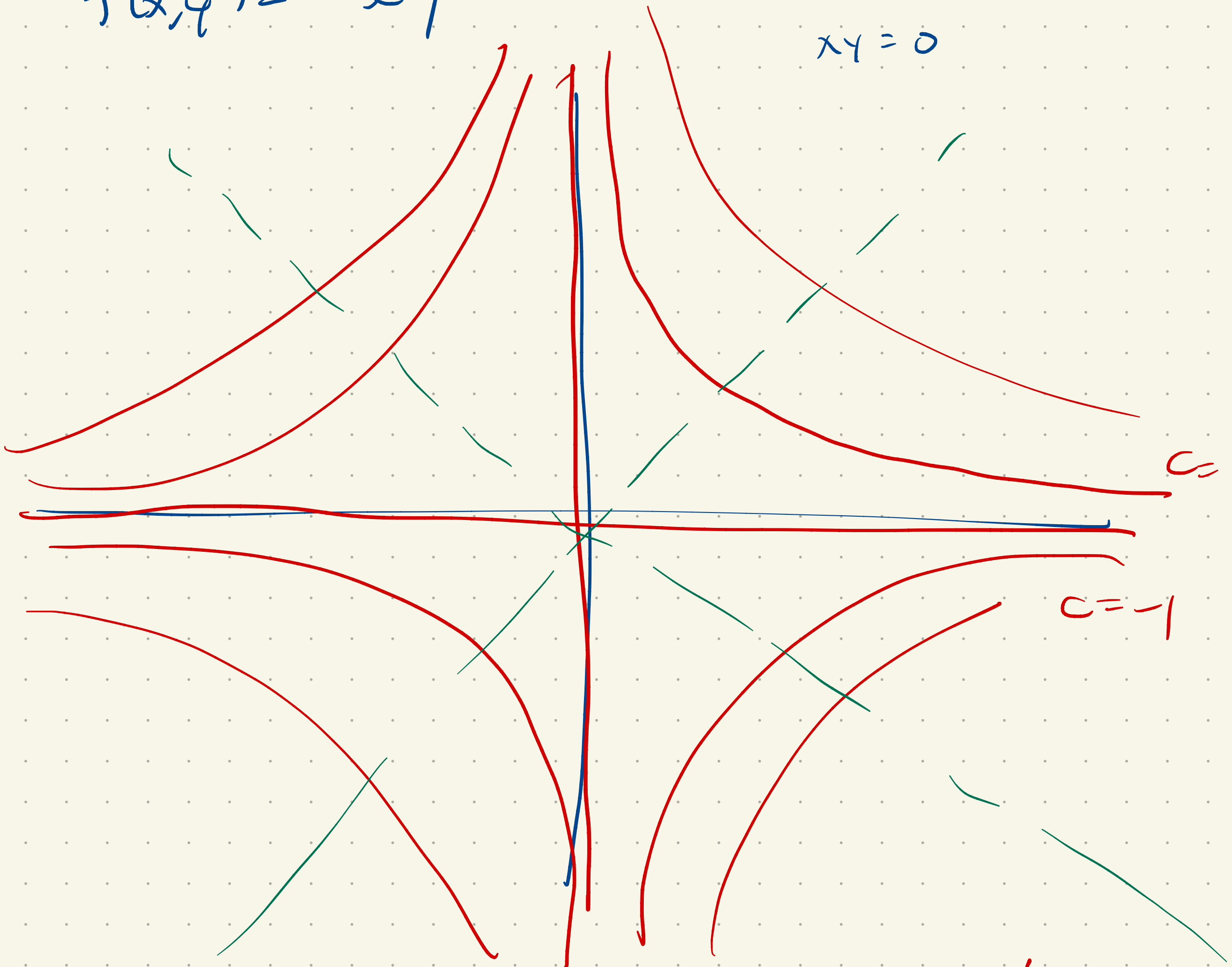


$f(x,y) = xy$

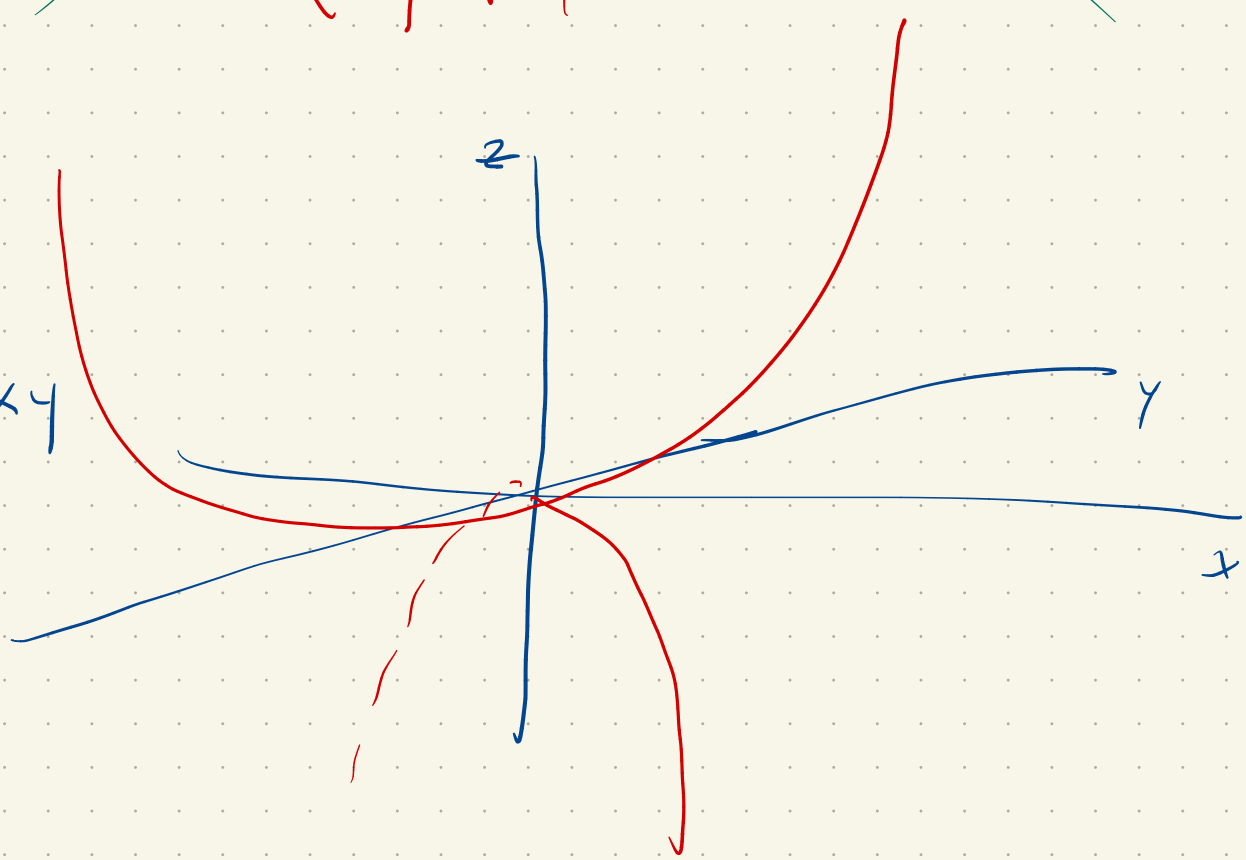
$xy = 1$

$xy = -1$ etc.

$xy = 0$



$z = xy$



We can also have functions of 3 variables. It's harder to graph them. (don't have the dims)

But we can still talk about level sets

$$F(x,y,z) = x^2+y^2+z^2$$

Level set r: sphere of radius r.