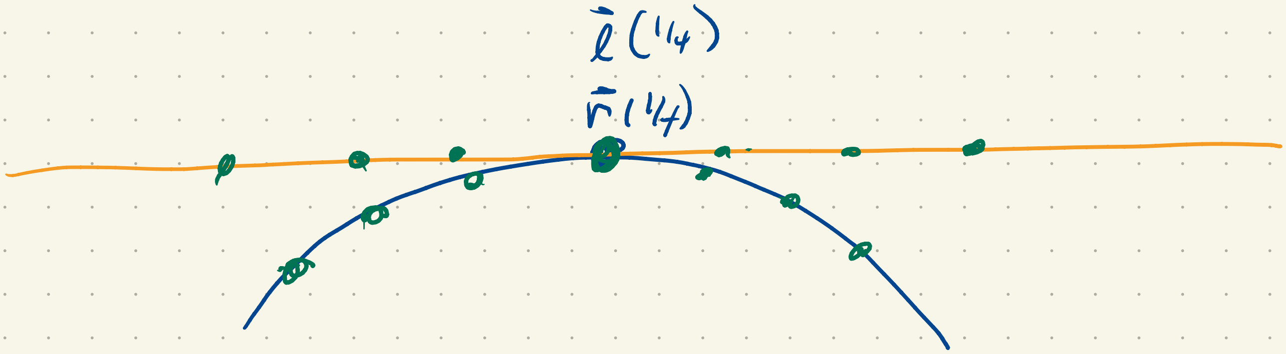
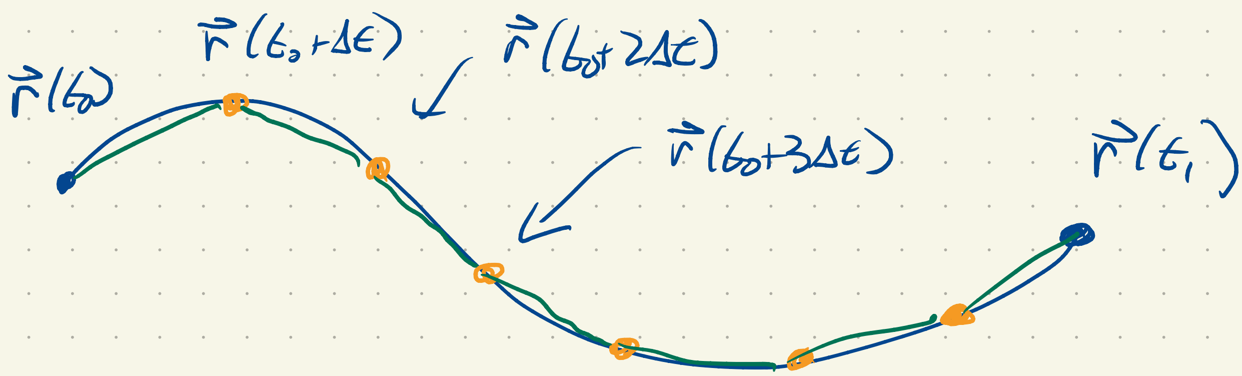


Arclength



Approximate by adding up lengths of line segments.

$$\Delta t = \frac{t_1 - t_0}{n}$$

First segment: $\|\vec{r}(t_0+\Delta t) - \vec{r}(t_0)\|$

Second $\|\vec{r}(t_0+2\Delta t) - \vec{r}(t_0+\Delta t)\|$

etc.

linear approx

Moreover, $\vec{r}(t_0+\Delta t) \approx \vec{r}(t_0) + \vec{r}\,'(t_0)\,\Delta t$

$$\left[\vec{r}\,'(t_0) \approx \frac{\vec{r}(t_0+\Delta t) - \vec{r}(t_0)}{\Delta t}\right]$$

def of derivative

$$\vec{r}(t_0+2\Delta t) \approx \vec{r}(t_0+\Delta t) + \vec{r}\,'(t_0+\Delta t)\,\Delta t$$

$$\vdots$$

$$\underbrace{\|\vec{r}(t_0+\Delta t) - \vec{r}(t_0)\|}_{\text{size of displacement}} \approx \underbrace{\|\vec{r}\,'(t_0)\|}_{\text{speed}}\,\underbrace{\Delta t}_{\text{length of time}}$$

$$\|\vec{r}(t_0+2\Delta t) - \vec{r}(t_0+\Delta t)\| \approx \|\vec{r}\,'(t_0+\Delta t)\|\,\Delta t$$

Approximate arclength: $\displaystyle\sum_{k=0}^{n-1} \|\vec{r}\,'(t_0+k\Delta t)\|\,\Delta t$

$n \to \infty$

arclength: $\displaystyle\int_{t_0}^{t_1} \|\vec{r}\,'(t)\|\,dt$

Units make sense $[\vec{r}\,'] = \frac{m}{s}$

$[dt] = s$ $\qquad \frac{m}{s} \cdot s = m$

length

Does this work?

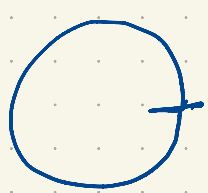
$\vec{r}(t) = \langle 5\cos(t), 5\sin(t) \rangle$

$t_0 = 0, \; t_1 = 2\pi$

$\vec{r}\,'(t) = 5 \langle -\sin(t), \cos(t) \rangle$

$\|\vec{r}\,'(t)\| = 5$

$$\int_0^{2\pi} 5 \, dt = 10\pi \;\leftarrow \text{arclength}$$



circumference: $2\pi \cdot 5 = 10\pi$ ✓

What about $\vec{r}(t) = \langle 5\cos(3t), 5\sin(3t) \rangle$?

Now $0 \le t \le \frac{2\pi}{3}$ covers whole circle

$$\|\vec{r}\,'\| = 5 \cdot 3 = 15$$

$$\int_0^{2\pi/3} 15\, dt = \frac{2\pi}{3} \cdot 15 = 10\pi \checkmark$$

$$\boxed{\text{In fact: Arclength is independent of parameterization}}$$

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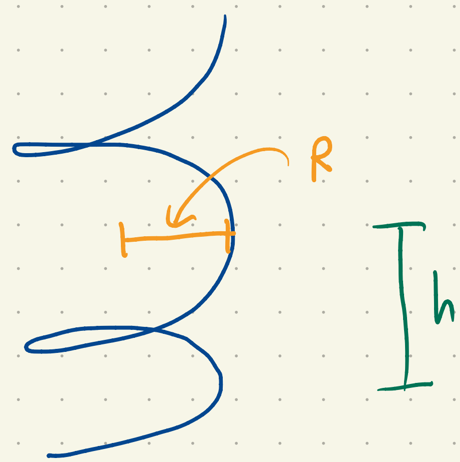
helix: 2 parameters



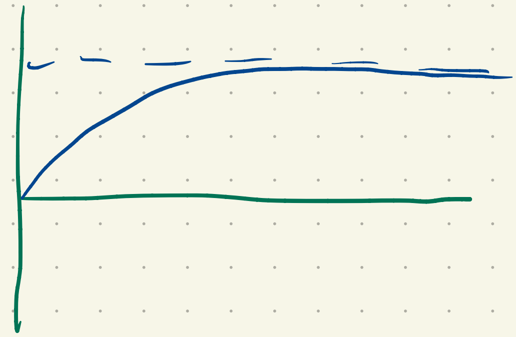
$$\vec{r}(t) = \left\langle R\cos\left(\tfrac{2\pi}{h}t\right), R\sin\left(\tfrac{2\pi}{h}t\right), t \right\rangle$$

$$\vec{r}(0) = \langle R, 0, 0 \rangle \qquad \vec{r}(h) = \langle R, 0, h \rangle$$

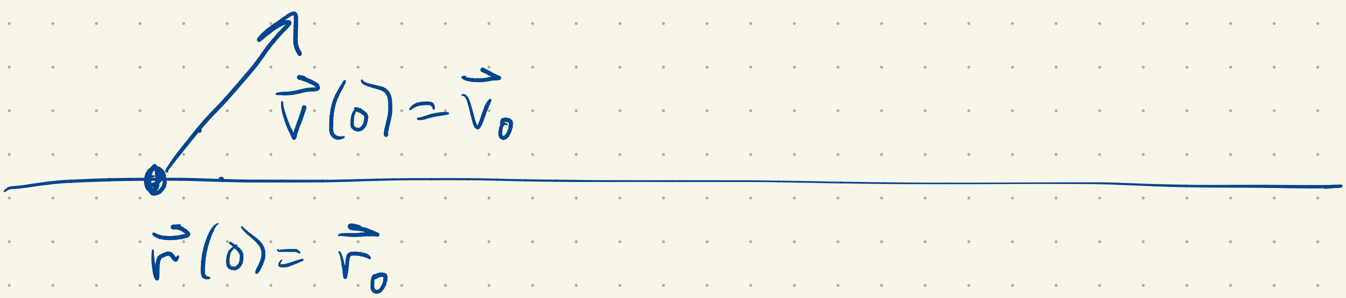
$V(t) = 50(1-e^{-t})\ \ell$

$V(0) = 0$

as $t \to \infty$ $\quad V(t) \to 50$



Section 3.4



Projectiles close to earth:

$$\vec{F}_g = -9.8\,\hat{k} \quad m/s^2$$

$$\vec{v}(t) = \int -9.8\,\hat{k}\,dt + \vec{C}_1$$

$$= -9.8t\,\hat{k} + \vec{C}_1$$

$$\vec{v}(0) = \vec{C}_1$$

$$\vec{v}(t) = -9.8t\,\hat{k} + \vec{v}_0$$

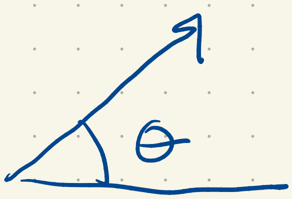
$$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{C}_2$$

$$\vec{r}(t) = \frac{-9.8t^2}{2}\hat{k} + \vec{v}_0 t + \vec{C}_2$$

$$\vec{r}(0) = 0 + 0 + \vec{C}_2$$

$$\vec{r}(t) = \vec{r}_0 + t v_0 - \frac{9.8}{2}t^2\hat{k}$$

($9.8 \to 0 \Rightarrow$ linear motion!)



$$\vec{r}_0 = \vec{0}$$

$$\vec{v}_0 = v_0\cos(\theta)\hat{i} + v_0\sin\theta\,\hat{k}$$

$$\vec{r}(t) = v_0\cos\theta\, t\,\hat{i} + \left[v_0\sin\theta\, t - \frac{9.8}{2}t^2\right]\hat{k}$$

This is a parabolic trajectory.

When is $z = 0$? $\quad t\left[v_0 \sin\theta - \frac{9.8}{2} t\right] = 0$

$t = 0$ or

$$t = \frac{2v_0}{9.8} \sin\theta$$

e.g. $V_0 = 150\,m/s \qquad \theta = \pi/4 = 45^\circ$

How far when strikes ground?

$$t = \frac{300}{9.8} \frac{1}{\sqrt{2}} \simeq 21.64$$

$$x = V_0 \cos\theta\, t$$

$$= 150 \frac{1}{\sqrt{2}} \frac{300}{9.8} \frac{1}{\sqrt{2}} = \frac{150 \cdot 150}{9.8} \simeq 2295\,m$$

Peak height?

$$\left[v_0 \sin\theta t - \frac{9.8}{2} t^2\right] = z(t)$$

$$z'(t) = v_0 \sin\theta - 9.8t$$

$$z'(t) = 0 \Rightarrow \quad t = \frac{v_0 \sin\theta}{9.8}$$

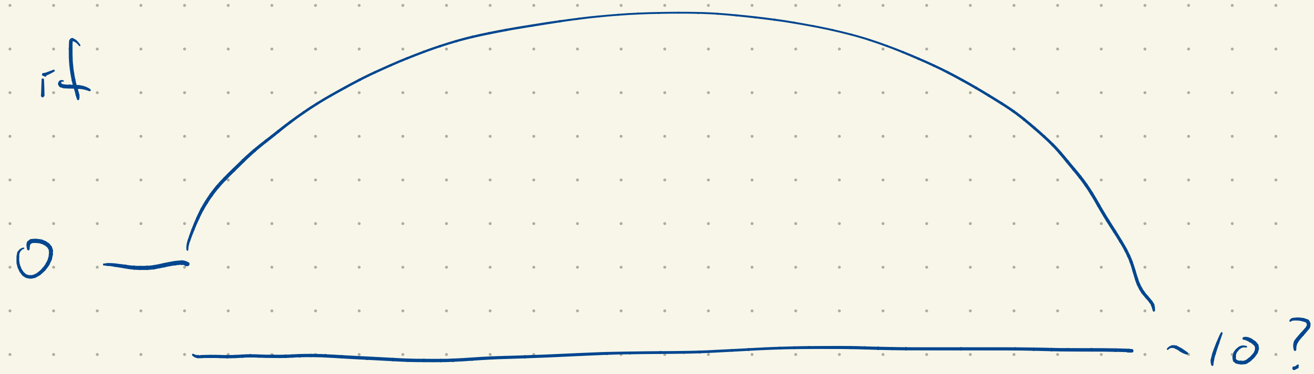
So peak happens at $t = \dfrac{150 \cdot \sin(45°)}{9.8} = 10.82\,s$

$z(10.82) = 574$ m

$$\frac{(v_0 \sin\theta)^2}{9.8} - \frac{9.8}{2} \frac{(v_0 \sin\theta)^2}{(9.8)^2}$$

$$= \quad \frac{1}{2} \frac{(v_0 \sin\theta)^2}{9.8} = 573$$

What if



$$t\left[v_0 \sin\theta - \frac{9.8}{2}t\right] = -10$$

$$-\frac{9.8}{2}t^2 + t\,\frac{150}{\sqrt{2}} + 10 = 0$$

$$t = 21.74$$

$$x \approx 2306$$