e.g. $\langle 1,2,3 \rangle \times \langle 3,1,2 \rangle$

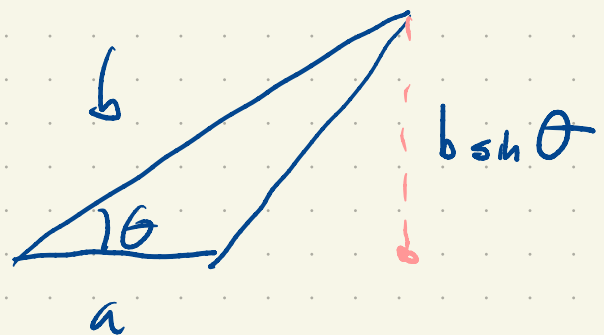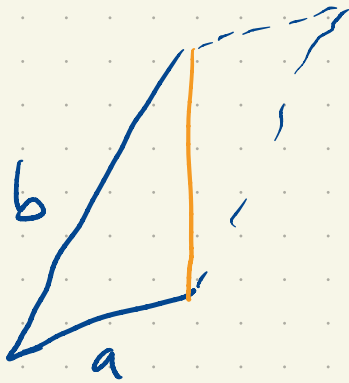$$\begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{array}$$

$$\hat{\imath} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= \hat{\imath}(4-3) - \hat{\jmath}(2-9) + \hat{k}(1-6)$$

$$= \hat{\imath} + 7\hat{\jmath} - 5\hat{k}$$

What is this quantity $\|\vec{a}\| \|\vec{b}\| \sin\theta$?

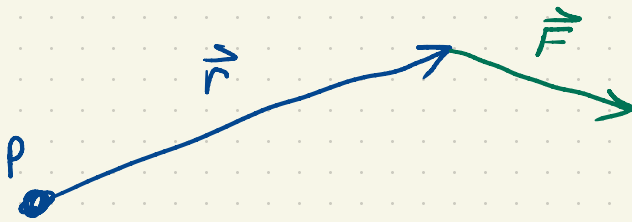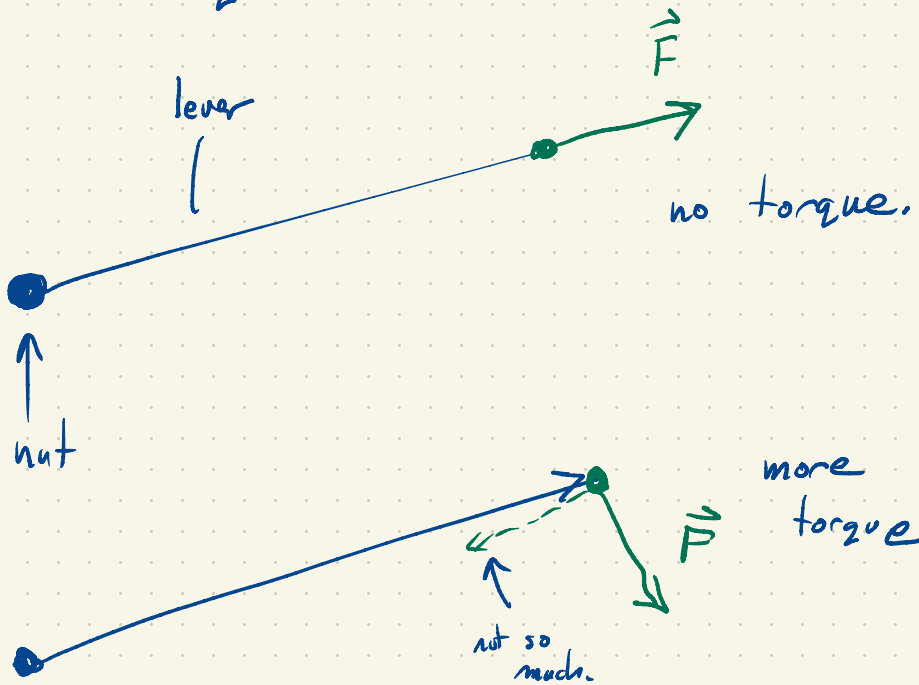
Area of triangle: $\frac{1}{2} ab \sin\theta$

$\|\vec{a}\| \|\vec{b}\| \sin\theta$ is the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

Application: Torque   (angular analog of force)

lever

$\vec{F}$

no torque.

↑
nut

more
torque

not so
much.

$\vec{r}$

$\vec{F}$

P

By definition: $\vec{\tau} = \vec{r} \times \vec{F}$ (Nm) $(ft\,lb)$
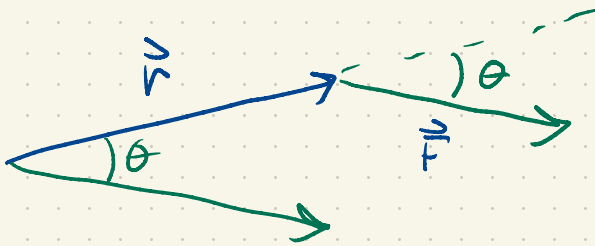
↑ torque of $\vec{F}$ at P.

Observe: If $\vec{F}$ is parallel to $\vec{r}$ $\vec{r} \times \vec{F} = 0$

$\Rightarrow$ zero torque.

Torque is a vector. Its direction encodes axis of rotation. Its length is the size of the angular force.
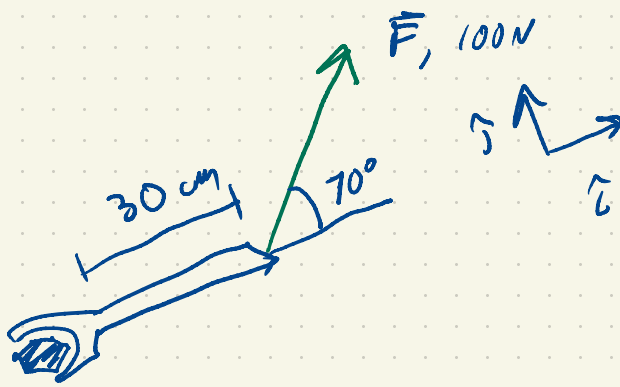
So, how long is $\vec{\tau}$?

$$\| \vec{r} \times \vec{F} \| = \| \vec{r} \| \ \| \vec{F} \| \ \sin\theta$$

$0 \le \theta \le \pi$

maximized at

$\theta = \frac{\pi}{2}$.

e.g.

$$\vec{F} = 100 \left(\cos 70° \, \hat{\imath} + \sin 70° \, \hat{\jmath} \right)$$

$$\vec{r} = 0.3 \, \hat{\imath}$$

$$\vec{r} \times \vec{F} = 0.3 \cdot 100 \cdot \hat{\imath} \times \left(\cos 70° \, \hat{\imath} + \sin 70° \, \hat{\jmath} \right)$$

$$= 30 \left(0 + \sin 70° \, \hat{k} \right)$$

$$= 30 \cdot \sin 70° \, \hat{k}$$

$$= 28.2 \, \hat{k} \; N_m.$$
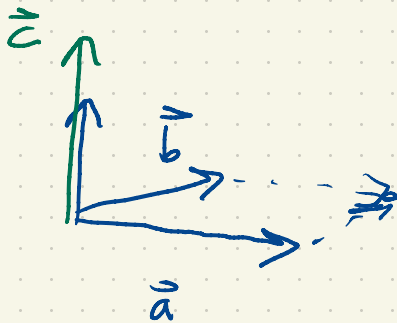
Convention: ⊙ ⊗

out of page

Scalar triple product

$$\vec{c} \cdot (\vec{a} \times \vec{b})$$

1) It's a number.

2) $\vec{c} = \vec{a}$, it's 0, $\vec{c} = \vec{b}$, it's 0

3) what if $\vec{c}$ is perp to $\vec{a} \times \vec{b}$?
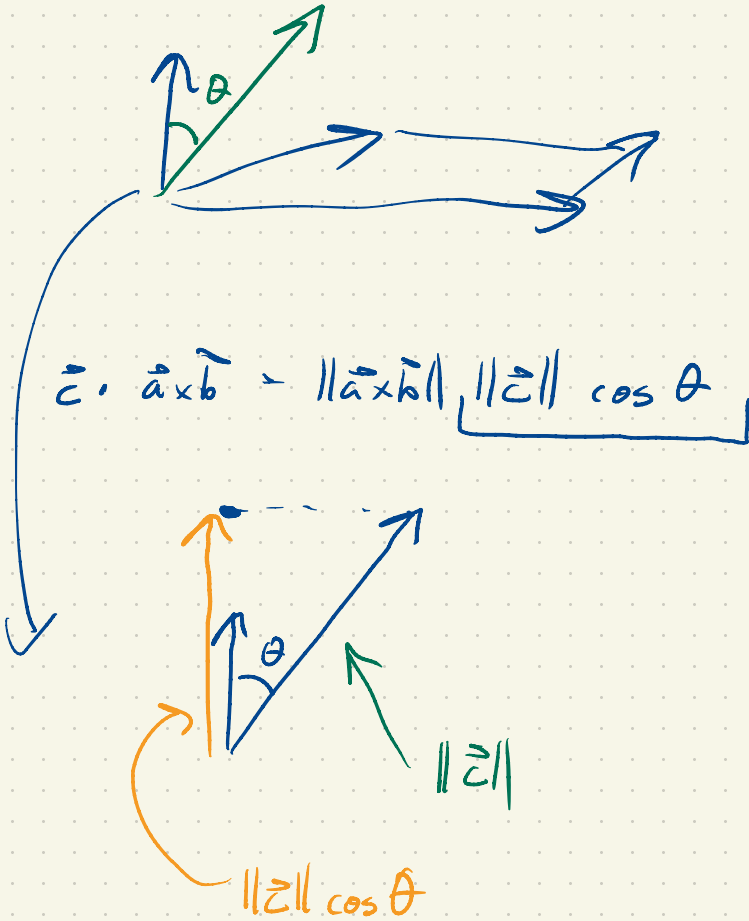

$$\vec{c} \cdot (\vec{a} \times \vec{b}) =$$

$$\|\vec{c}\| \, \|\vec{a} \times \vec{b}\| = vol.$$

It's positive if

$(\vec{a}, \vec{b}, \vec{c})$ are right handed.

4) If $\vec{c}$ is in plane spanned, it's 0. (volume)

5) general case

$$\vec{c} \cdot \vec{a} \times \vec{b} = \|\vec{a} \times \vec{b}\| \underbrace{\|\vec{c}\| \cos \theta}$$

$\|\vec{c}\|$

$\|\vec{c}\| \cos \theta$

It's a sum the area of parallelepiped.