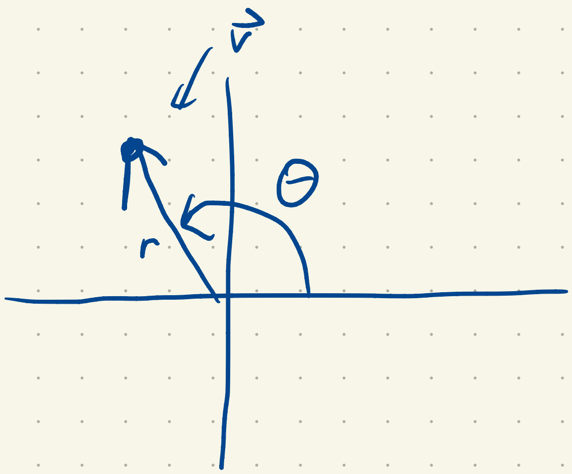
Vectors in the plane can be described by

1) length $r = |\vec{v}|$

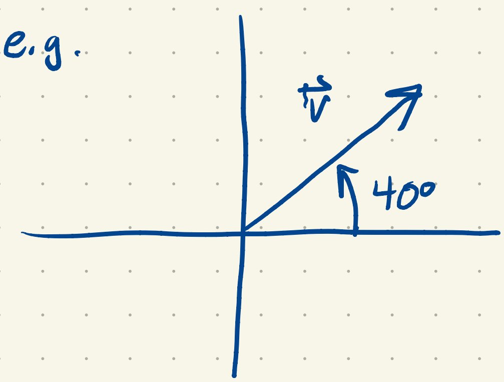
2) angle θ relative to the x-axis
↑ signed



$$\vec{v} = r\overbrace{\langle \cos\theta, \sin\theta \rangle}^{\text{unit vector}}$$

$$= \langle r\cos\theta, r\sin\theta \rangle$$

e.g.



$|\vec{v}| = 10$ km/h

$\vec{v} = v_1\hat{i} + v_2\hat{j}$

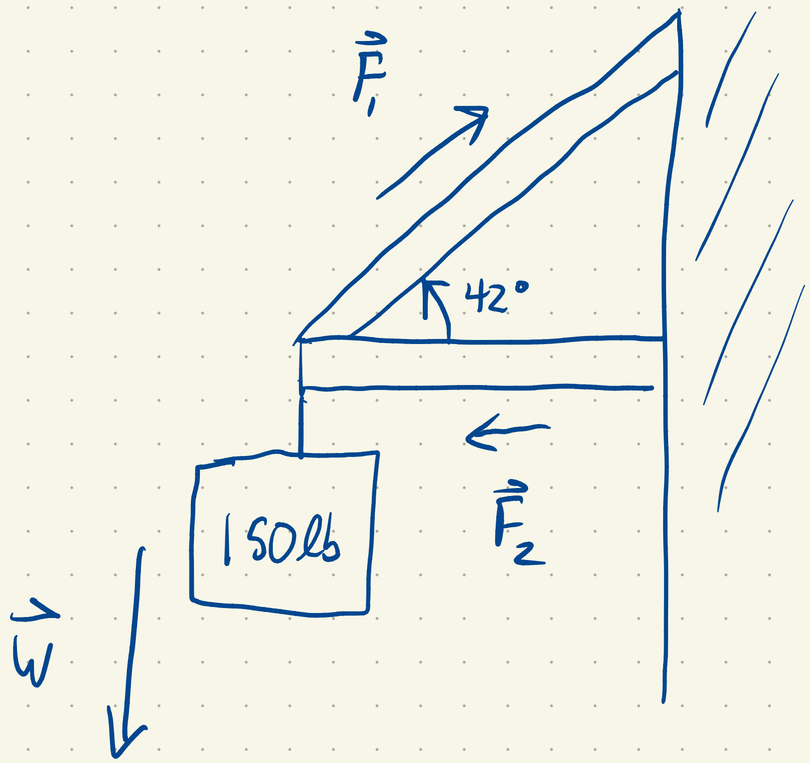
$v_1 = 10\cos 40° \approx 7.7$ (km/h)

$v_2 = 10\sin 40° \approx 6.4$ km/h

$\vec{v} = 7.7\hat{i} + 6.4\hat{j}$ km/h

When multiple forces act on an object,

the net force is the vector sum of the forces.



Total force $\vec{W} + \vec{F}_1 + \vec{F}_2$.

Static equilibrium: no acceleration, net force is zero.

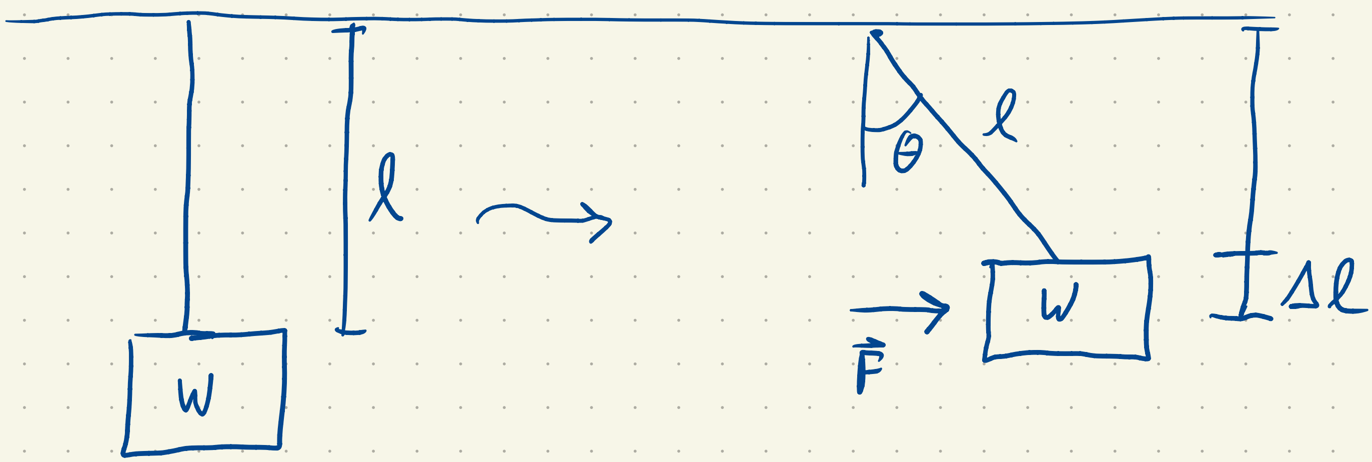
$$\vec{W} = -150\,\hat{j}\,, \quad \vec{F}_2 = -F_2\,\hat{i} \quad F_2 = |\vec{F}_2|, \text{ unknown.}$$

$$\vec{F}_1 = F_1 \cos 42^\circ\,\hat{i} + F_2 \sin 42^\circ\,\hat{j}$$

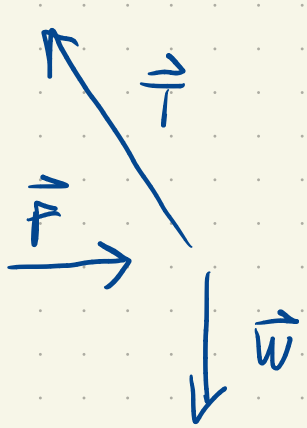
$$\vec{W} + \vec{F}_1 + \vec{F}_2 = \underbrace{\left(F_1 \cos 42^\circ - F_2\right)}_{0}\hat{i} + \underbrace{\left(F_2 \sin 42^\circ - 150\right)}_{0}\hat{j}$$

$$F_2 = 150/\sin 42° = 224 \text{ lb}$$

$$F_1 = F_2/\cos 42° = 301.7 \text{ lb}$$



On homework: given $\vec{F}$, $\theta = ?$, $\Delta \ell = ?$



$$\vec{F} + \vec{W} + \vec{T} = 0 \quad !$$

Two equations,

two unknowns $(|\vec{T}|, \theta)$

$\rightsquigarrow (\theta, \Delta \ell)$

2.3 The Dot Product

Def: $\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Multiply two vectors, get a scalar.

This depends on having picked coordinates.

But: $\vec{a} \cdot \vec{a} = a_1^2 + a_2^2 + a_3^2 = |\vec{a}|^2$.

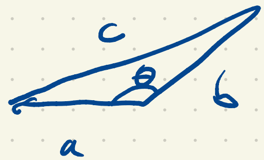
This depends only on the length scale.

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + |\vec{b}|^2$$

$$\vec{a} \cdot \vec{b} = \frac{1}{2}\left[|\vec{a}|^2 + |\vec{b}|^2 - |\vec{a} - \vec{b}|^2 \right]$$

it depends <u>only</u> on the length scale.

Law of cosines:



$$2ab\cos\theta = a^2 + b^2 - c^2$$

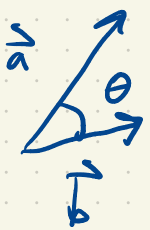
$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$$

$0 \le \theta \le \pi$

$\uparrow$

$\theta = 0$, done

$\theta = \pi$, easy



$$\vec{i}\cdot\vec{j} = 1\cdot 0 + 0\cdot 1 + 0\cdot 0 = 0$$

$$\vec{j}\cdot\vec{k} = 0\cdot 0 + 1\cdot 0 + 0\cdot 1 = 0$$

$$\vec{k}\cdot\vec{i} = 0\cdot 1 + 0\cdot 0 + 1\cdot 0 = 0$$

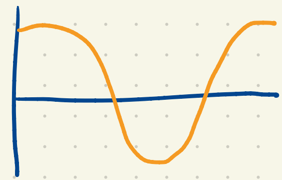
$\cos\theta = 0$ in each case

$\Rightarrow \theta = \pi/2 \Rightarrow \perp$, perp

Fundamental Property of Dot Product:

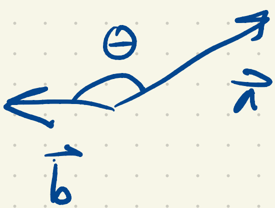
$$\vec{a} \cdot \vec{a} = |\vec{a}|^2$$

$$\vec{a} \cdot \vec{b} = 0 \quad \text{iff} \quad \vec{a} \perp \vec{b}$$



$\vec{a} \cdot \vec{b} > 0$: $\cos\theta > 0$, acute

$\vec{a} \cdot \vec{b} < 0$: $\cos\theta < 0$, obtuse



E.g. Find the angle in radians between

$\vec{a} = \langle 1, 2, 3 \rangle$

$\vec{b} = \langle -1, 2, 1 \rangle$

$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$

$\vec{a} \cdot \vec{b} = -1 + 4 + 3 = 6$

$|\vec{a}|^2 = 1 + 4 + 9 = 14$

$|\vec{b}|^2 = 1 + 4 + 1 = 6$

$$\cos\theta = \frac{6}{\sqrt{6}\sqrt{14}} = \sqrt{6/14}$$

$$\theta = \arccos\left(\sqrt{6/14}\right)$$

$$= 0.857 \text{ rad}$$

$$= 49.1^\circ$$