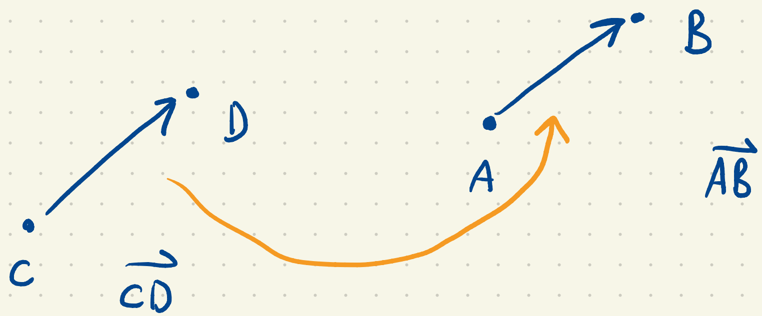
Sections 2.1, 2.2

Displacement Vectors

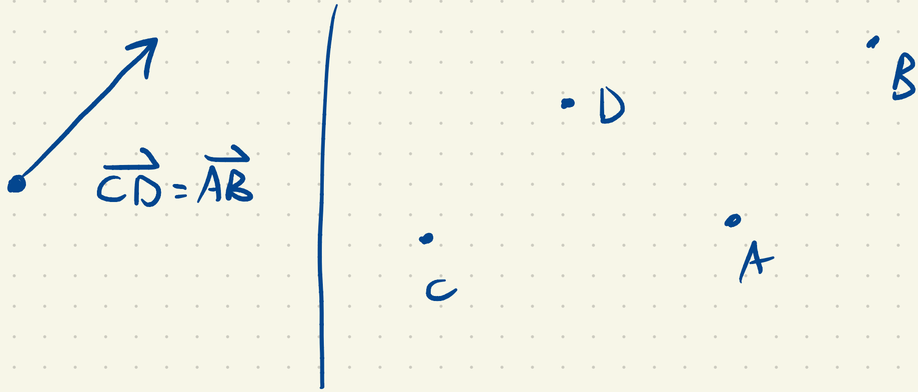


The displacements from C to D and from A to B are the same.

If we translate C to A then D lands on A.

We identify:

$$\overrightarrow{CD} = \overrightarrow{AB}$$



Land of vectors
(displacements)

Euclidean space
(points)

Displacement vectors have a direction (mostly) and a length.

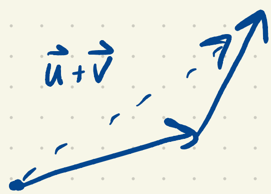
$|\vec{AB}|$ is just the distance from A to B.

$|AB|$

The zero vector does not have a direction.

Operations on vectors:

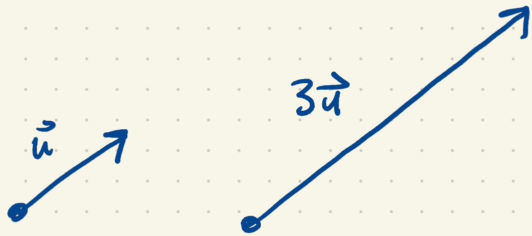
1) Vector addition



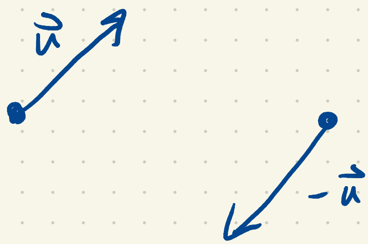
$$\vec{AB} + \vec{BC} = \vec{AC}$$

2) Scalar multiplication: $a > 0$, $\vec{u} \neq 0$

$a\vec{u}$ is the vector parallel to $\vec{u}$ with length $a|\vec{u}|$



$a<0$: points in opposite direction



$a \cdot \vec{0} = \vec{0}$ no matter what a is.

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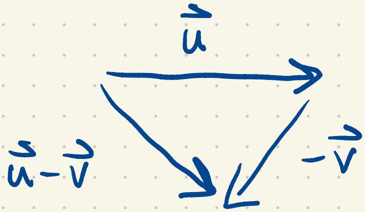
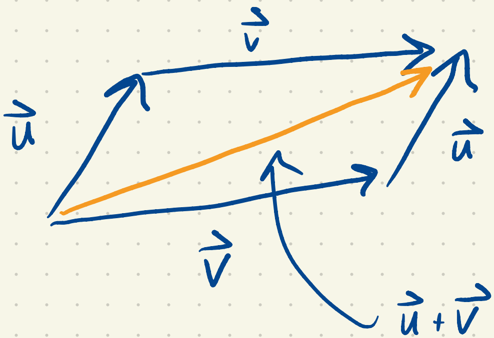
3) Subtraction

$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$



Addition is commutative



The zero vector is special:

$\vec{0}+\vec{u} = \vec{u}+\vec{0}$ no matter what $\vec{u}$ is.

Note: the <u>origin</u> of your Cartesian coordinate system is arbitrary.

The zero vector (zero displacement!) is a very real thing.

Once you establish Cartesian coordinates, displacement vectors gain coordinates as well:

$$P = (x_0, y_0, z_0)$$
$$Q = (x_1, y_1, z_1)$$

$$\overrightarrow{PQ} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

$\hookrightarrow$ not standard, but used in text

It's just the difference in coordinates.

The geometric vector operations have very natural algebraic equivalents:

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

Properties:

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

$$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$$

$$(c+d)\vec{a} = c\vec{a} + d\vec{a}$$

$$c(d\vec{a}) = (cd)\vec{a}$$

$\vec{a} + 0 = \vec{a}$ | $1\vec{a} = \vec{a}$

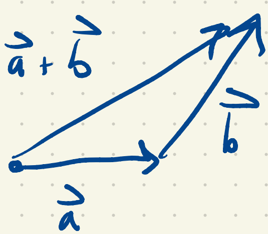
The length of a vector is the Euclidean length of the displacement

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right)$$

$$|c\vec{a}| = |c|\,|\vec{a}|$$

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \text{ ?}$$ *Nope.*



$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

Common operations:

$$\vec{u} = \langle \sqrt{5}, 2, 4 \rangle$$

$$|\vec{u}|^2 = 5 + 4 + 16 = 25$$

$$|\vec{u}| = 5$$

$$\frac{1}{5}\vec{u} = \left\langle \frac{\sqrt{5}}{5}, \frac{2}{5}, \frac{4}{5} \right\rangle$$

$$\uparrow$$

$$\left|\frac{1}{5}\vec{u}\right| = \left|\frac{1}{5}\right| |\vec{u}| = \frac{1}{5} \cdot 5 = 1$$

We say $\frac{1}{5}\vec{u}$ is a unit vector.

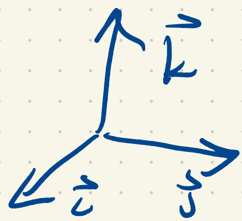
It points parallel to $\vec{u}$ but has unit length.

We give names to three unit vectors that point along the coordinate axes:

$$\vec{i} = \langle 1,0,0 \rangle$$

$$\vec{j} = \langle 0,1,0 \rangle$$

$$\vec{k} = \langle 0,0,1 \rangle$$



(standard basis vectors)

These depend on your coordinates.

O is special. $\vec{i}$ is <u>not</u>.

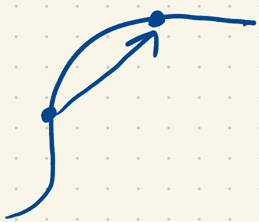
$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$= a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}.$$

Other vectorial quantities

- velocity (m/s)
- acceleration (m/s^2)
- force $(kg\ m/s^2 = N)$ (lb ☹)

The other parts are "decoration"



$$\frac{\overbrace{P(t_1) - P(t_0)}^{\text{vector}}}{t_1 - t_0} \qquad \frac{\text{dist}}{\text{time}}$$

send $t_1 \to t_0$ and get an instantaneous velocity.

All the rules thus far also apply to these physical variations of displacement vectors.