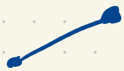
Pre sections 2.1–2.2

Coordinates.

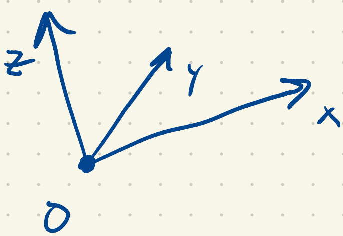
How to make Cartesian coordinates in 3 dimensions

1) pick an origin, O

2) pick a unit distance  (has a notion of distance!)

3) pick 3 mutually perpendicular rays through origin, and label x, y, z

(has a notion of perpendicular!)



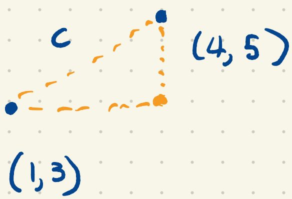
4) The triple $(1, 2, -3)$ encodes the point obtained by

- move in x direction 1 unit
- move in y direction 2 units
- move in $-z$ direction 3 units.

These coordinates are linked to the geometry of 3-d Euclidean space.

There are other systems you might want to use in other applications, but these are a convenient default.

Distance between two points:



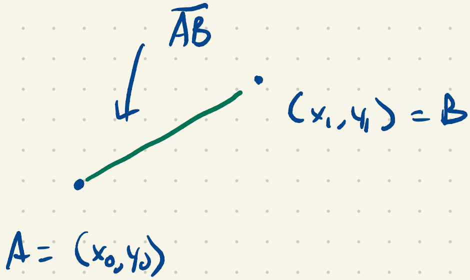
$5-3=2=b$

$4-1=3=a$

$$a^2+b^2=c^2$$

$$3^2+2^2=c^2$$

$$9+4=c^2 \implies c=\sqrt{13}$$



$\Delta x = x_1 - x_0$

$\Delta y = y_1 - y_0$

$dist^2 = \Delta x^2 + \Delta y^2$

In 3-d:

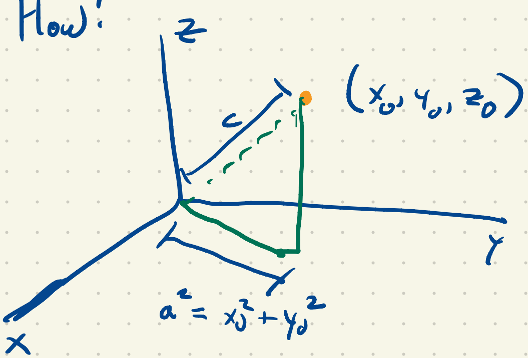
(x_1, y_1, z_1)

$\Delta z = z_1 - z_0$

(x_0, y_0, z_0)

$$dist^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$
$$= (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2$$

How?

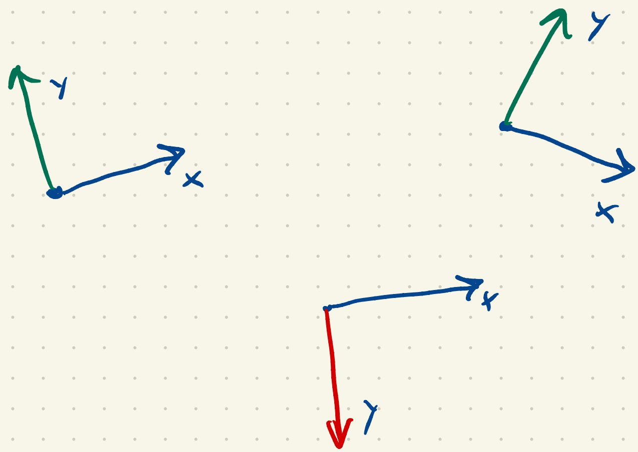


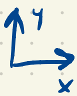
$a^2 = x_0^2 + y_0^2$

$$c^2 = a^2 + z_0^2 = x_0^2 + y_0^2 + z_0^2$$

Orientation.

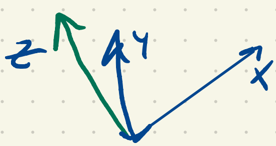
Planes have two classes of cartesian coordinates

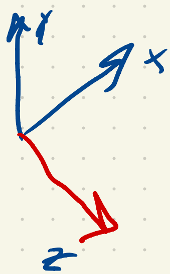


Probably the 

ones feel more familiar.

An analogous phenomenon in 3-d.





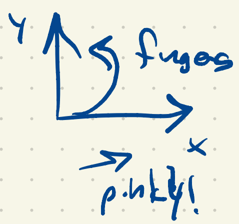
Can't drag one onto other.

If the thing you are coordinatizing has right hands in it, we prefer right-handed coordinate systems.

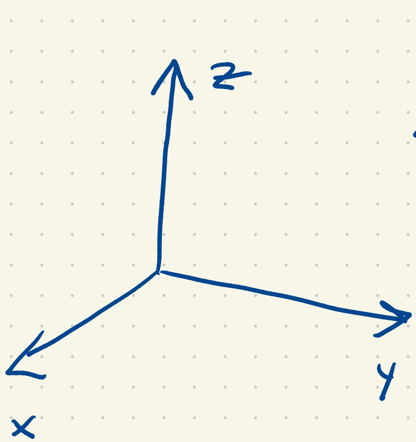
a) use right hand (critical!)

b) lay pinky along positive x-axis

c) rotate hand until fingers curl in direction of positive y-axis



d) thumb points along positive z-axis.



a right handed system

Spheres

The sphere of radius r centered at

$P = (x_0, y_0, z)$ is the set of points

(x, y, z) satisfying

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$