Transformers, an Introduction

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Previously, on LLMs for the People

- Words and bits of text are represented as tokens.
- An LLM is a (deterministic) map from a sequence of tokens to a probability distribution over tokens.
- The probability distribution predicts the next token that follows the sequence.
- The GPT-2 (2019) and GPT-3 (2022) processing pipelines have three phases:
    1. Embedding tokens to latent space
    2. A stack of transformers
    3. Final stage conversion to a probability distribution.

Tokens

Input text is broken into a sequence of tokens:

Can	adians	are	friendly	and	approach	able	.

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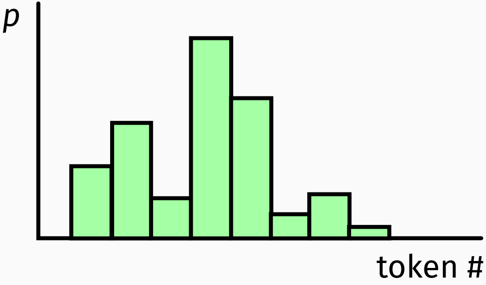
- Roundtrip unicode text to token sequence to unicode text is lossless.
- GPT-2 has a vocabulary of roughly 50000 tokens.
- GPT-3.5: 100000 tokens.

The basic function

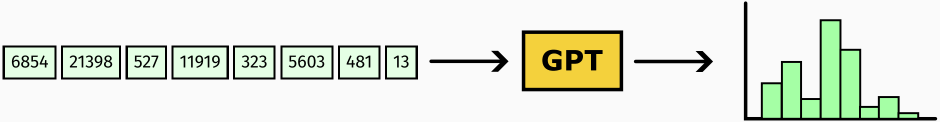
GPT-2 is a function, depending on a very large number of numerical parameters.

Input: A sequence of up to 1024 tokens (the **context window**).

Output: A probability distribution over tokens.



The Iteration



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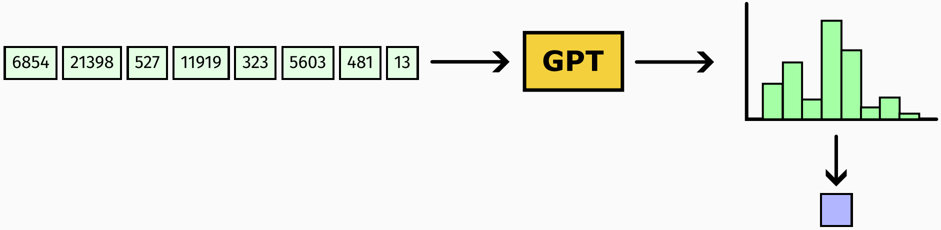

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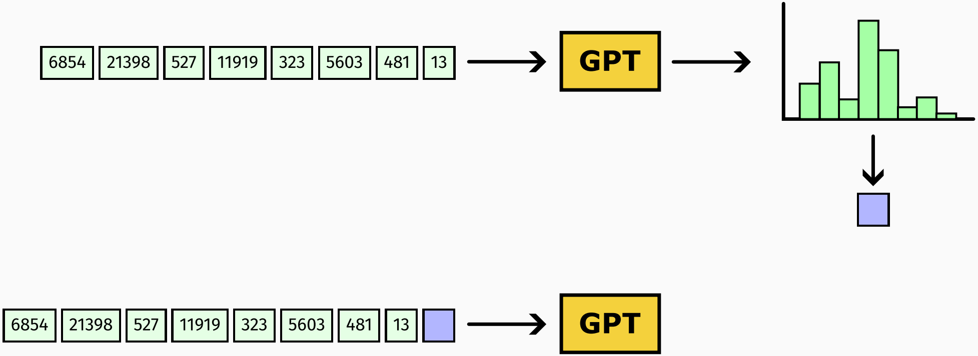

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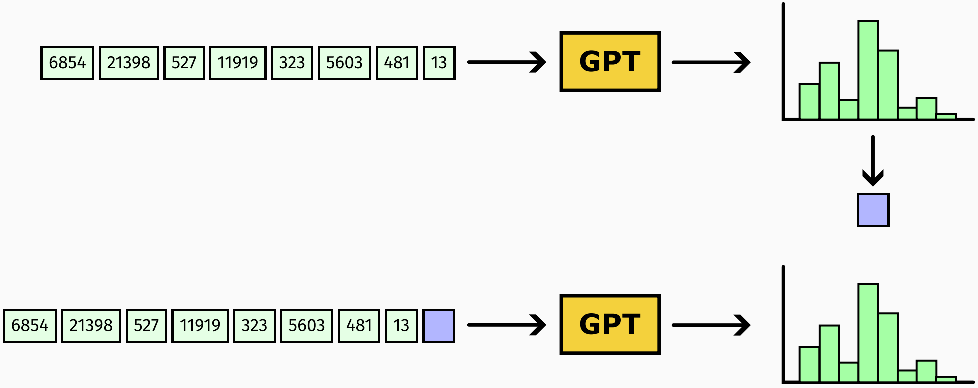

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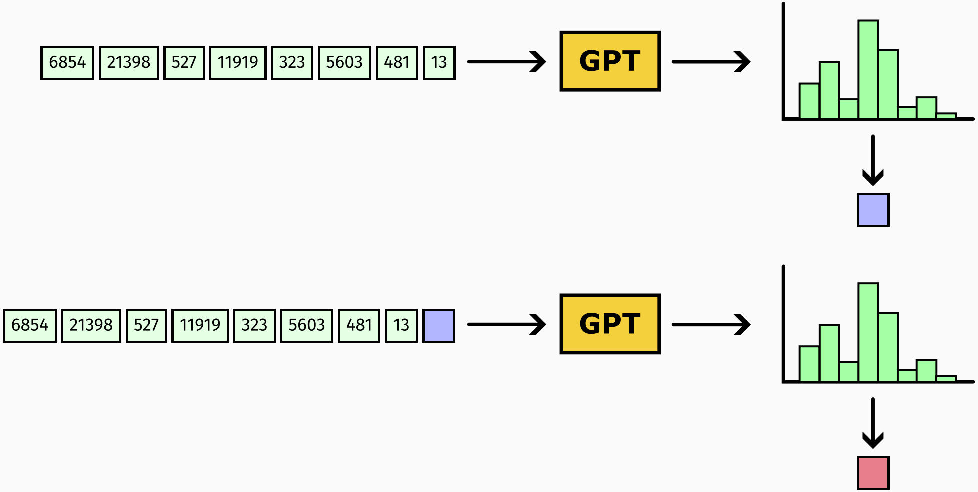
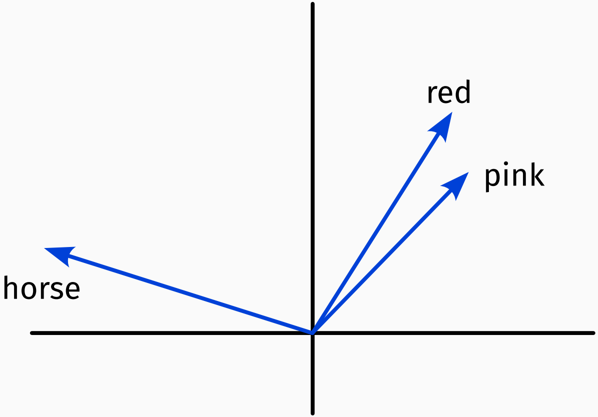


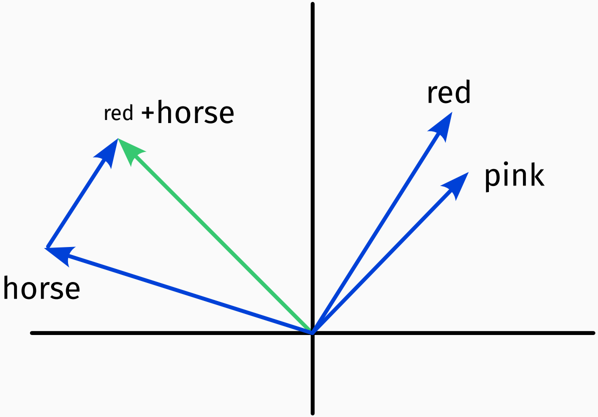
Latent Space

Tokens are immediately converted to vectors.

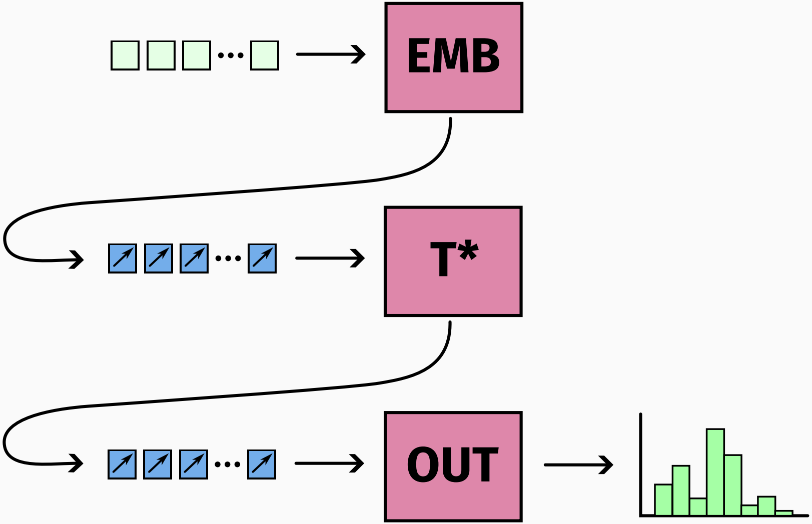


Latent Space

Tokens are immediately converted to vectors. (Actual dimension: 768 for GPT-2)



Main Pipeline

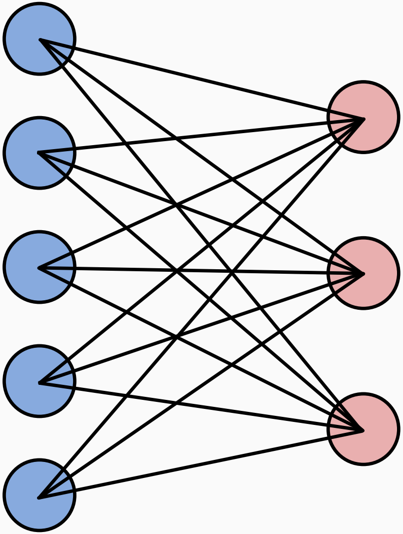


Ingredient: Linear Maps

A map $f : \mathbb{R}^n \to \mathbb{R}^m$ is linear if:

$$f(\mathbf{x} + \mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y})$$
$$f(c\mathbf{x})a = cf(\mathbf{x})$$

for all inputs $\mathbf{x}$ and $\mathbf{y}$ and all numbers c.



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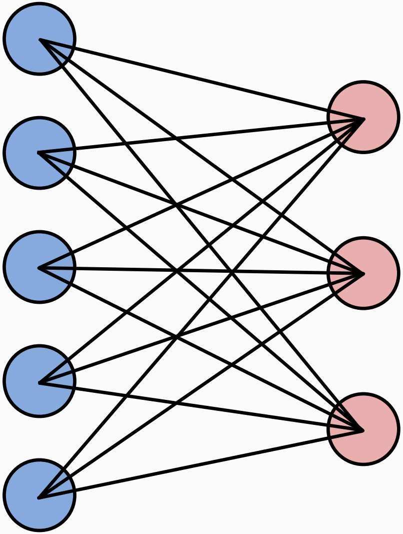
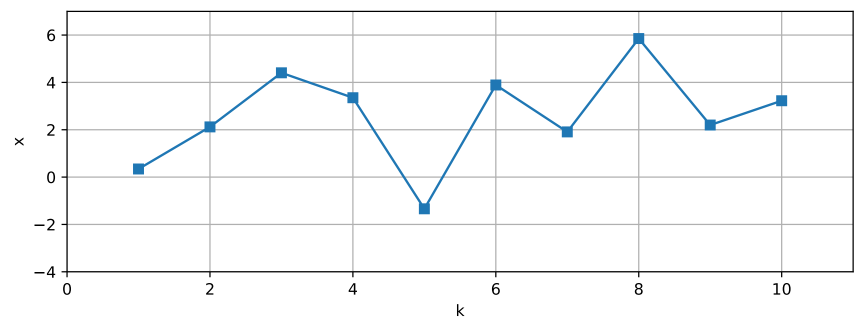
- We can represent such a map via a collection of $n \cdot m$ weights
- GPUs are great at computing these



Ingredient: Weight and Balance

“Weight and balance” steps keep vectors in latent space at a reasonable size.



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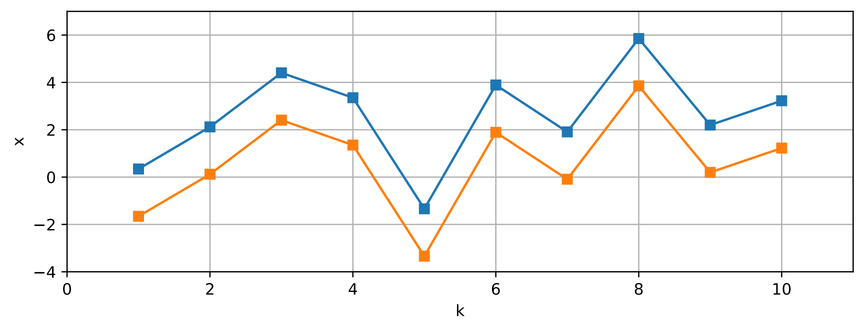
"Weight and balance" steps keep vectors in latent space at a reasonable size.

1. Remove the mean



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"Weight and balance" steps keep vectors in latent space at a reasonable size.

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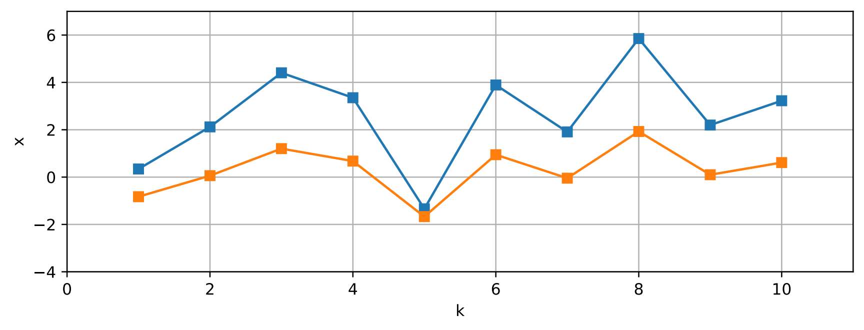
2. Scale to unit variance



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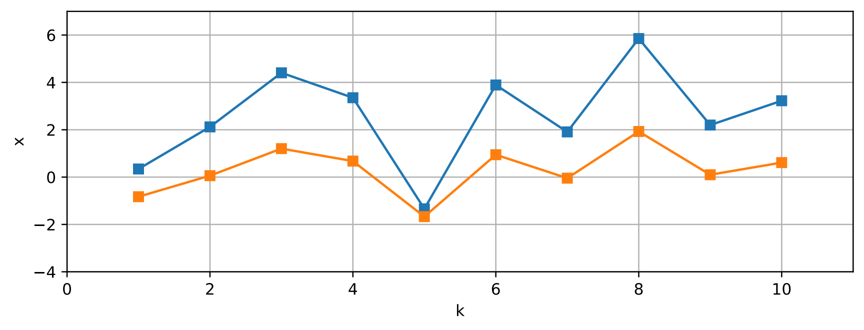
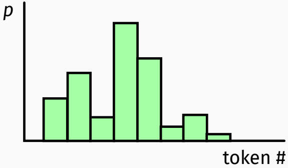


3. Training: new scale, new "zero vector"

Ingredient: softmax

The final output is a probability distribution:

$$(p_1, p_2, \ldots, p_{50000}), \quad p_i \geq 0, \quad \sum_{i=1}^{50000} p_i = 1$$



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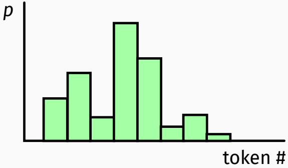

softmax

1. Start with arbitrary $(w_1, w_2, \ldots, w_{50000})$

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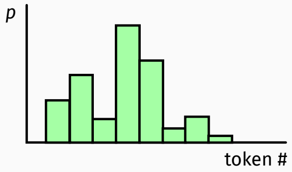

softmax

1. Start with arbitrary $(w_1, w_2, \ldots, w_{50000})$
2. Make $(q_1, q_2, \ldots, q_{50000})$ with $q_i = e^{w_i}$
   - Observe $q_i \geq 0$

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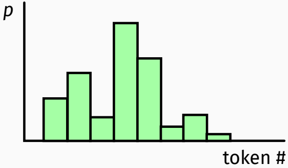


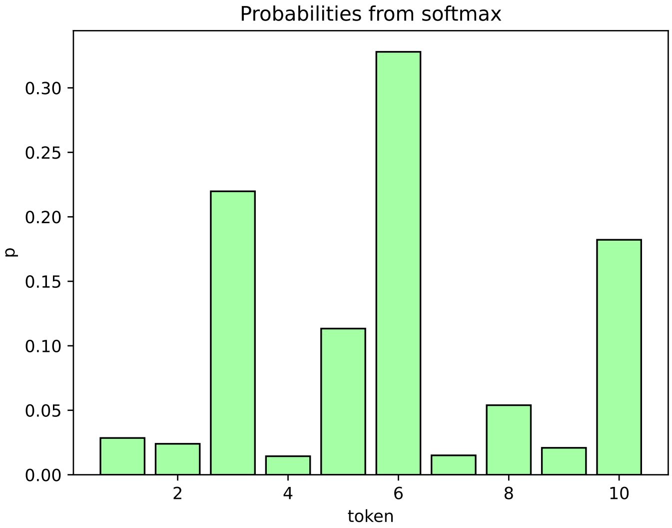
softmax

1. Start with arbitrary $(w_1, w_2, \ldots, w_{50000})$
2. Make $(q_1, q_2, \ldots, q_{50000})$ with $q_i = e^{w_i}$
   - Observe $q_i \geq 0$
3. Let $q_{\text{total}} = q_1 + q_2 + \cdots + q_{50000}$
4. Then $p_i = q_i / q_{\text{total}}$

Ingredient: softmax

Scale matters:

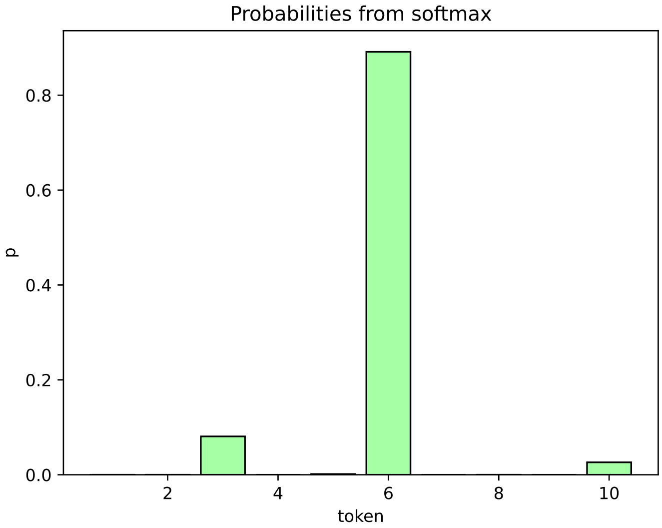
$$x \to p$$



Ingredient: softmax

Scale matters:

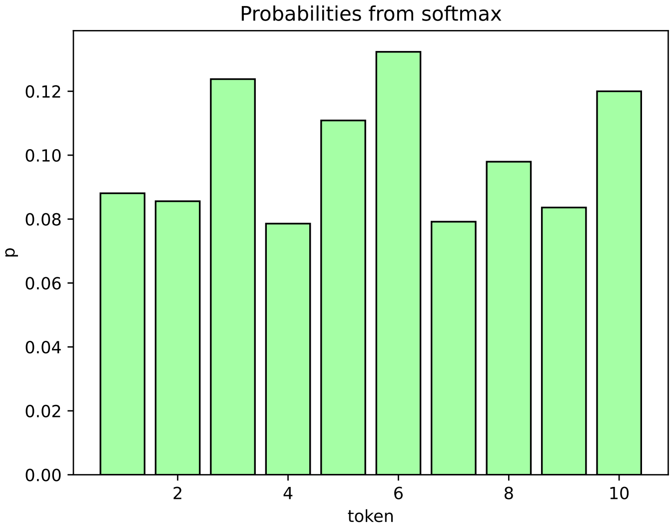
$$6x \to p$$



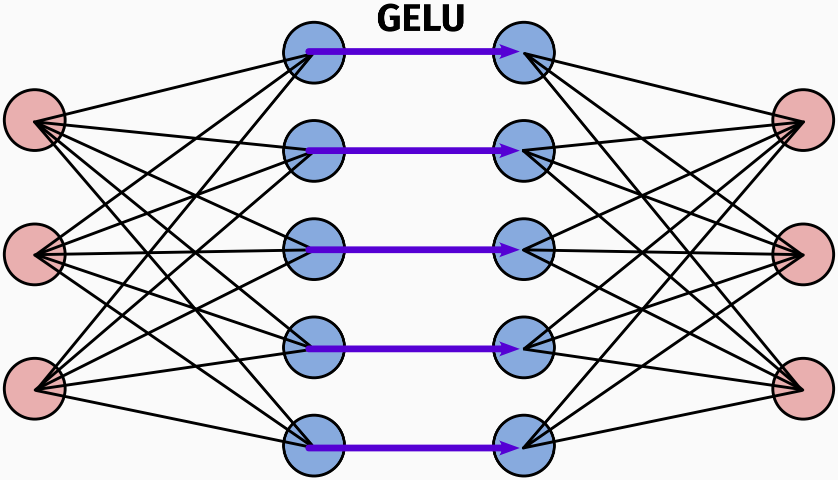
Ingredient: softmax

Scale matters:

$$x/6 \to p$$



Ingredient: Thin Neural Net (Feedforward Layer)



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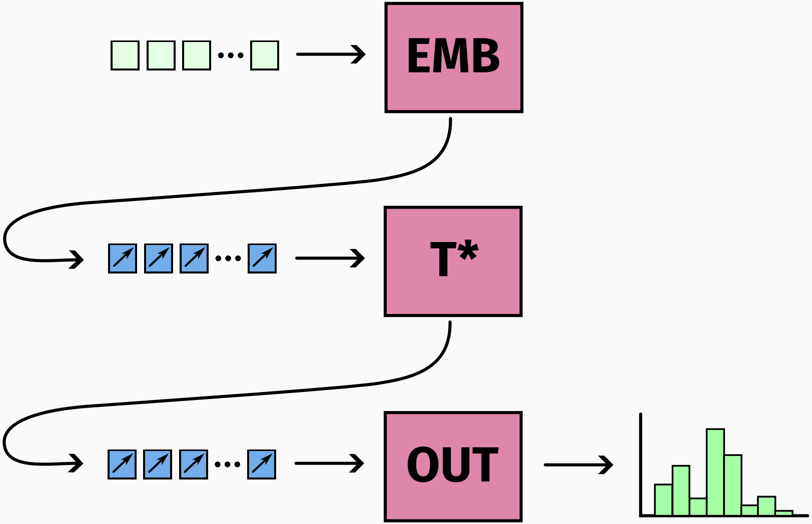
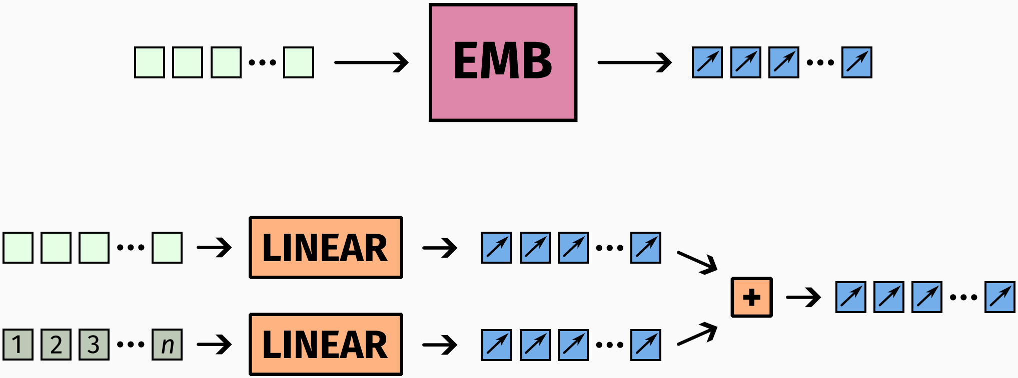
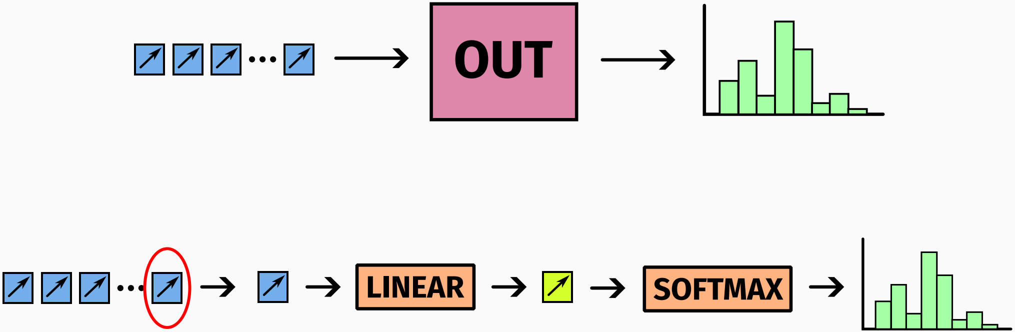


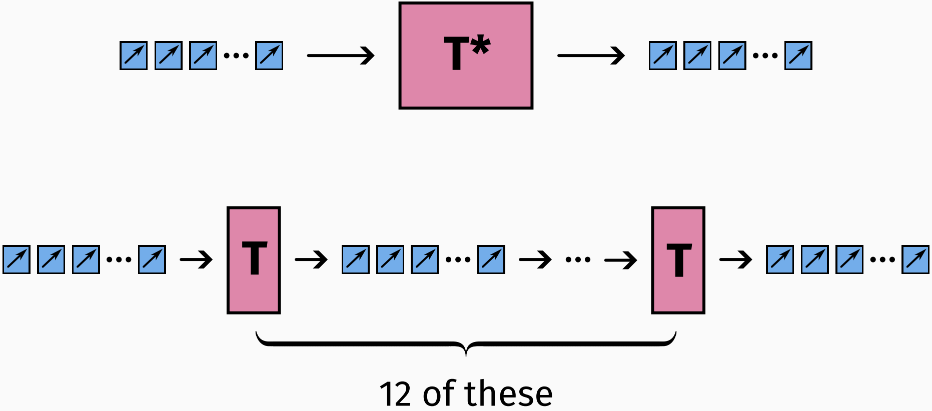
Token Embedding



Final Output



Stack of Transformers



Motivation for Attention

- Natural language translation
- Distant information needs to be associated

I get up at 6:30 in the morning.

Morgens stehe ich um halb sieben auf.

(for additive attention)

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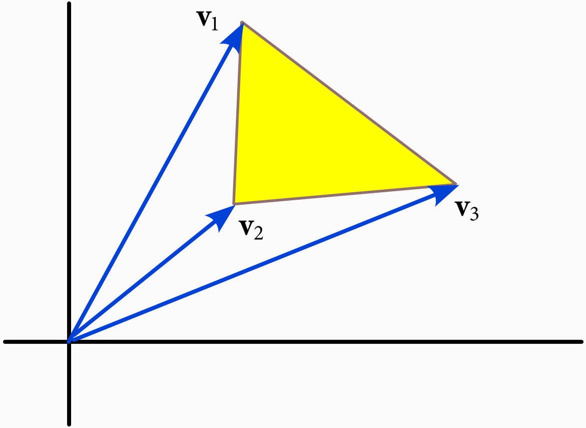
Bahdanau et. al, Neural Machine Translation by Jointly Learning to Align and Translate, 2014. (for additive attention)

Combining information = convex combinations



$$\mathbf{v} = b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3$$

$$0 \le b_i \le 1$$

$$b_1 + b_2 + b_3 = 1$$

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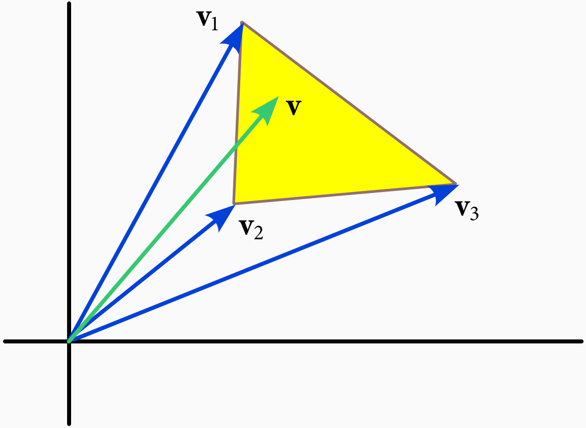

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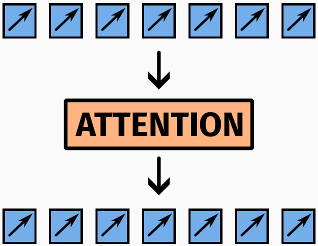
$$0 \leq b_i \leq 1$$

$$b_1 + b_2 + b_3 = 1$$

Attention Module

Morgens stehe ich um halb sieben auf



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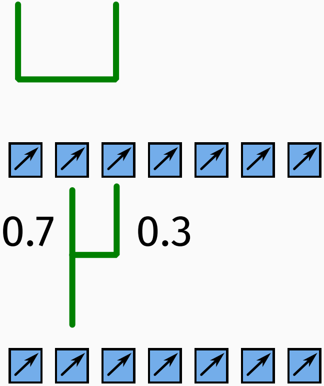

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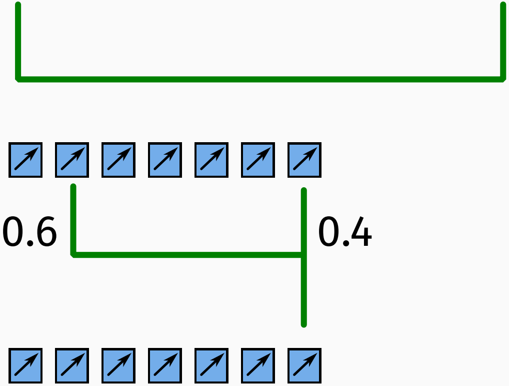

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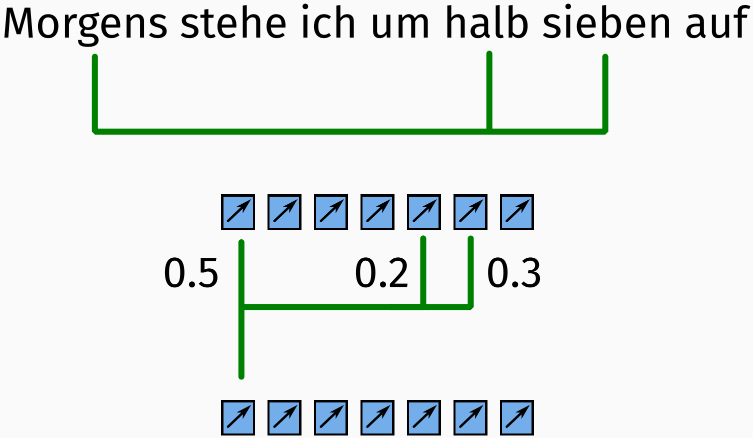
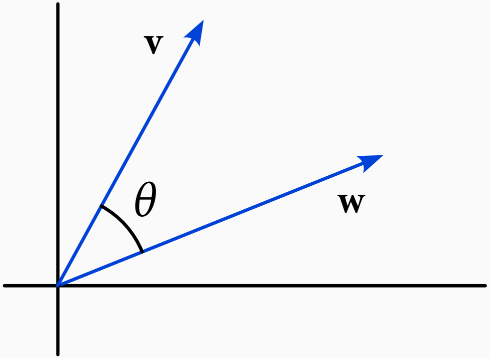


Determination of Weights I

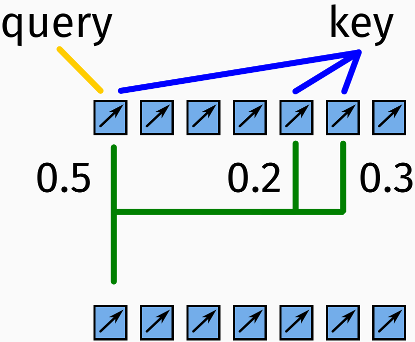
Dot product measures alikeness of vectors.

$$\mathbf{v} \cdot \mathbf{w} = ||\mathbf{v}||\mathbf{w}|| \cos\theta$$
$$= v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$



Determination of Weights II

Computing the weights a_{ij} where key j contributes to query i:



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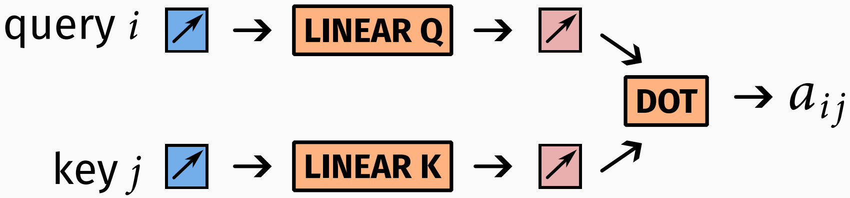

Determination of Weights III

The weights so far don’t make a convex combination.

- Use softmax: $a_{ij} \to b_{ij}$ to ensure $0 \leq b_{ij} \leq 1$ and $\sum_j b_{ij} = 1$

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The weights so far don't make a convex combination.

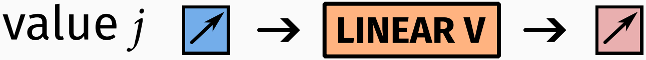
- Use softmax: $a_{ij} \to b_{ij}$ to ensure $0 \leq b_{ij} \leq 1$ and $\sum_j b_{ij} = 1$

And scaling matters:

- Use softmax: $a_{ij}/\sqrt{768} \to b_{ij}$

Projections

We don’t actually make convex combinations of the original vectors.



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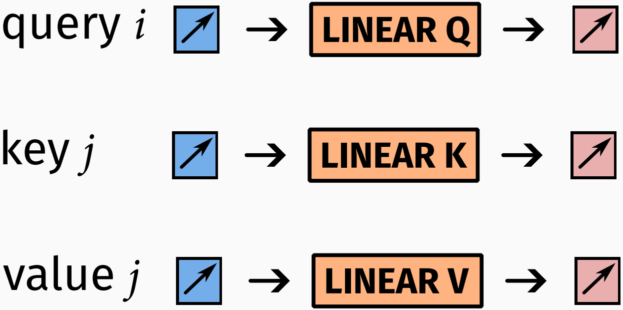

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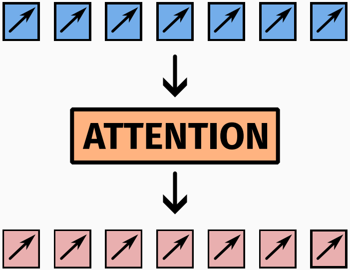

Causality

One more detail: each slot can only use information from the past

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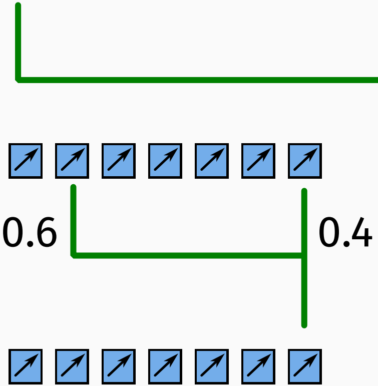

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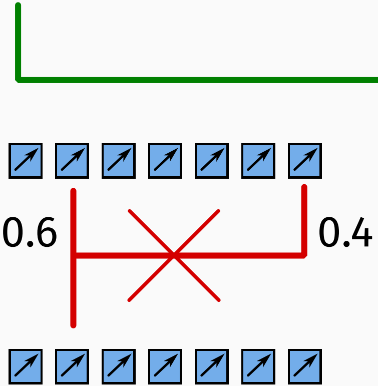
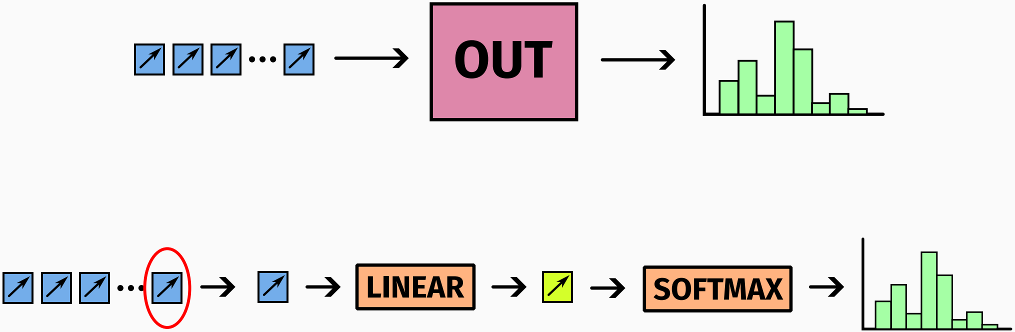


The Truth about OUT



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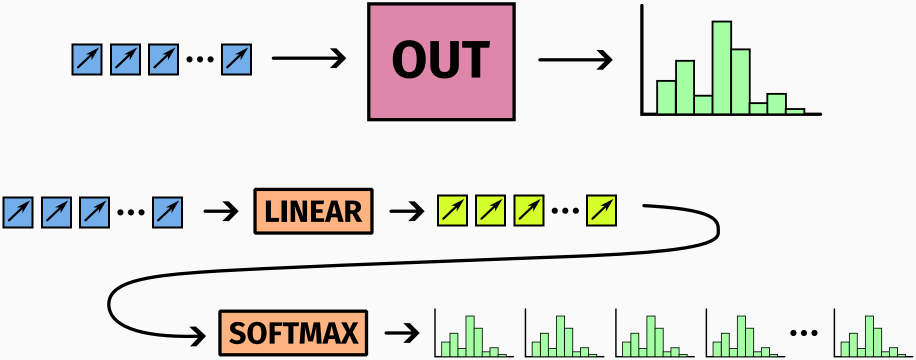
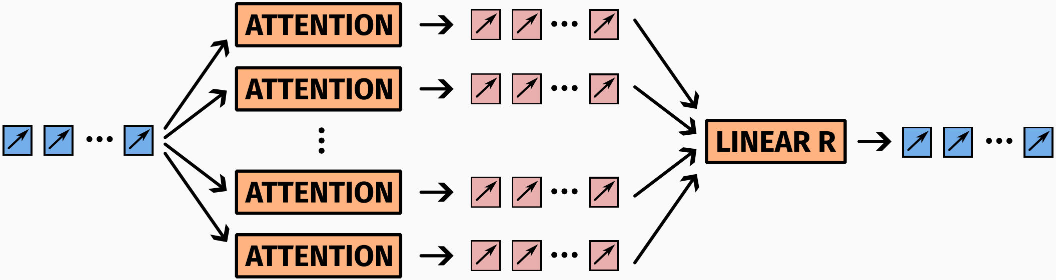


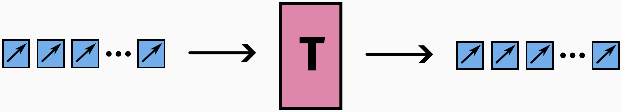
Multihead Attention

Increased parallelism by having more than one attention block happen at the same time.



A Single Transformer

Attention is All You Need, Vaswani et. al., 2017



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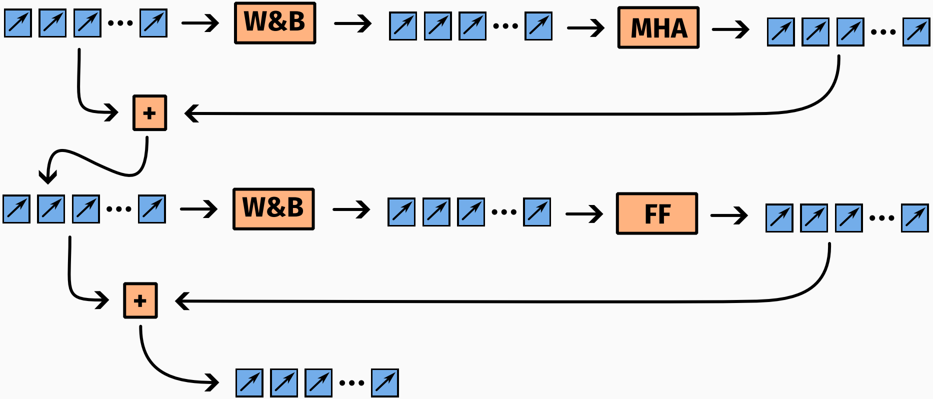

Summary

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- The map is deterministic, but selection of tokens from the distribution can be probabilistic.
- The entire stream of tokens is reprocessed from scratch to generate the next token.
- The inner machinery is implemented entirely out of familiar mathematical maps:
  - Linear maps
  - Dot products, scaling, vector addition
  - Element-wise activation functions
  - softmax

Fin

Thank you!

Parameter Counts

GPT-2: ~125M parameters

1. EMB and OUT linear maps: 40%
2. Feed forward 45%
3. Attention linear maps: 15%

Parameter Counts

GPT-3: ~175B parameters

1. EMB and OUT linear maps: 20%
2. Feed forward 60%
3. Attention linear maps: 20%