Math F651: Homework 9 Due: April 5, 2026

1. Suppose $\langle x \rangle_{\alpha \in A}$ is a net in X that does not converge to $x \in X$. Show that there is an open set U containing x and a subnet $\langle x_{\alpha_{\beta}} \rangle_{\beta \in B}$ such that $x_{\alpha_{\beta}} \notin U$ for all $\beta \in B$. Hint: For a particular 'bad' U, take B to be the entire subset of A such that $x_{\beta} \notin U$ and show that B is directed. Then show that there is a natural increasing cofinal map from B to A.

- **2.** Crossley 6.1 Show that the spaces [0,1] and (0,1) are homotopy equivalent by finding an explicit homotopy equivalence and its inverse between the two spaces.
- 3. Crossley 6.4

Suppose $f: X \to S^n$ is continuous and not surjective. Show that it is homotopic to a constant map.

4. Crossley 6.5

Show by means of an explicit homotopy that the map $f: S^1 \to S^1$ given by f(x, y) = (-x, -y) is homotopic to the identity.

- **5.** Show that a space X is contractible if and only if [X, X] consists of a single element.
- **6.** Lee 7-1