Note: The book has Exercises, which are interspersed among the prose, and Problems, which appear at the ends of the chapters. It can be easy to confuse the two. Exercises are denoted in blue.

1. Although quotient maps must take saturated open sets to open sets and saturated closed sets to closed sets, a quotient map need not be open or closed. The point of this exercise is to see an example.

Let A be the set of points (x, y) is $\mathbb{R}^2$ with $y = 0$ or $x \geq 0$. Let $\pi(x, y) = x$. Show that $\pi : A \to \mathbb{R}$ is a quotient map, but that it is neither open nor closed.

2. Let $\pi : X \to Y$ be a quotient map and let $A \subseteq X$ be a saturated closed set or a saturated open set. Show that $\pi|_A : A \to \pi(A)$ is a quotient map.

3. Problem 3-14

4. Problem 3-16

Your solution should not be longer than a page. Extra credit for the shortest correct solution.

5. No rigor please on this problem. Just coherent explanations, and maybe a picture or two.

a) Define an equivalence class on $\mathbb{C}$ where $z \sim w$ if there is $u \in S^1$ with $z = wu$. The quotient space $\mathbb{C}/\sim$ is a familiar topological space. Name it. (“is” means “is homeomorphic to, of course”).

b) Define an equivalence class on $\mathbb{R}^2$ where $(x, y) \sim (x + 1, -y)$ (along with all the relations then implied by transitivity). The resulting quotient space is a familiar one. Name it.

6. Let $X = \mathbb{R} \times \{0, 1\}$ and define an equivalence relation on X by $(0, 0) \sim (0, 1)$. Rigorously show that $X/\sim$ is homeomorphic to the union of the x- and y-axes in the plane.

7. Problem 4-1

8. **Exercise 4-4**