1. 4-23 (a,b)

2. The point of this exercise is to settle some details from the proof of the Brower fixed point theorem. We suppose $f : \mathbb{B} \to \mathbb{B}$ is continuous and that f does not have a fixed point.

a) Prove that for all $x \in \mathbb{B}^2$ there exists a unique $t(x) \in [1, \infty)$ such that $f(x)+t(x)(x-f(x)) \in S^1$.

b) Define

$$r(x) = f(x) + t(x)(x - f(x)),$$

so $r : \mathbb{B} \to S^1$. The graph of r is a subset of $\mathbb{B} \times S^1$. We wish to show that r is continuous, and since S^1 is compact and Hausdorff it is enough to show that the graph of r is closed. Do so. Hint: Suppose $(x_n, r(x_n)) \to (x, z) \in \mathbb{B} \times S^1$. Now show that $z = r(x)$. We'll discuss in the problem session what a boon the closed graph theorem is here.

3. Crossley 6.6: Show that if $f, g : S^1 \to S^1$ then $[f \circ g] = [g \circ f]$.

4. Lee 8-5

5. Lee 8-7