Name:

1. Suppose we want to find a polynomial $p(t) = c_1 + c_2 t$ passing through the three points with (t, y) coordinates given by $(-1, 0)$, $(1, 3)$ and $(2, 4)$. This can't be done, of course. Nevertheless, set up a system of the form $Ac = b$ to solve for the coefficients $c = (c_1, c_2)$. Your answer will consist of a 3×2 matrix A with numerical entries and a 3-vector b also with numerical entries.

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

2. Now set up the normal equation used to solve for the least squares solution. You do **not** need to solve the system. Your answer will be in the form $Bc = d$ where B is a matrix with numerical entries and d is a vector with numerical entries.

$$\underbrace{A^TA}_{B}\, c = \underbrace{B^Tb}_{d} \leftarrow \text{normal equation.}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$$

$$d = B^Tb = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$$

3. (Extra credit) Solve the system.

$$B^{-1} = \frac{1}{14}\begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1}\begin{bmatrix} 7 \\ 11 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 20 \\ 19 \end{bmatrix} = \begin{bmatrix} 10/7 \\ 19/14 \end{bmatrix}$$