Name:

1. The matrix

$$A = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & -4 \\ 2 & 7 \end{bmatrix}$$

admits the QR factorization $A = QR$ with

$$Q = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

You don't need to show this. Instead, use the QR factorization to solve $Ax = b$ with

$$b = \sqrt{5}\begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$$w = Q^T b = \frac{1}{\sqrt{5}}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}\sqrt{5}$$

$$= \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$Rx = \begin{bmatrix} 7 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$3x_2 = 1 \Rightarrow x_2 = 1/3$$

$$x_1 + 2x_2 = 7$$

$$\Rightarrow x_1 = 7 - \frac{2}{3} = \frac{19}{3}$$

$$x = \frac{1}{3}\begin{bmatrix} 19 \\ 1 \end{bmatrix}$$

2. Find two different left inverses for the matrix

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Want: $XA = 1$

$X = [1 \ 0 \ 0]$

$X = [0 \ \frac{1}{2} \ 0]$

also: $X = [0 \ 0 \ \frac{1}{3}]$

$X = [-2 \ 0 \ 1]$

and many more!