Name: Solutions

**1.** Suppose $B$ is a $7 \times 3$ matrix. Consider the block matrix:

$$A = \begin{bmatrix} I & 0 \\ B^T & I \end{bmatrix}.$$

Determine the dimensions of each identity matrix and the zero matrix. Be sure in your answer to distinguish between the two identity matrices.

$$\begin{bmatrix} 7 \times 7 & 7 \times 3 \\ 3 \times 7 & 3 \times 3 \end{bmatrix}$$

**2.** Let $a_1 = (1, -1)$ and $a_2 = (2, 0)$. I've done the Gram Schmidt algorithm on these two vectors and have determined that

$$q_1 = \frac{1}{\sqrt{2}}(1, -1), \quad q_2 = \frac{1}{\sqrt{2}}(1, 1).$$

I also found that $\tilde{q}_1 = (1, -1)$ and $\tilde{q}_2 = (1, 1)$ along the way.

a) Now let

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}.$$

Observe that the columns of $A$ are exactly $a_1$ and $a_2$. Compute the $QR$ factorization to determine matrics $Q$ and $R$ where $Q$ has orthonormal columns and $R$ is upper triangular such that

$$A = QR$$

$$Q = [q_1 \; q_2] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$\|\tilde{q}_1\| = \sqrt{2}$$

$$\|\tilde{q}_2\| = \sqrt{2}$$

$$R = \begin{bmatrix} \|\tilde{q}_1\| & q_1^T a_2 \\ 0 & \|\tilde{q}_2\| \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} \end{bmatrix}$$

$$q_1^T a_2 = \frac{1}{\sqrt{2}}(1,-1)^T(2,0) = \frac{1}{\sqrt{2}} \cdot 2 = \sqrt{2}$$

b) Verify that your previous answer worked. That is, multiply $Q$ and $R$ and show that you recover $A$.

$$QR = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \sqrt{2} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \checkmark$$