Name: Solutions

1. In this problem we represent a tiny 2×2 image by a vector $x = (x_1, x_2, x_3, x_4)$ of pixel intensities as follows:

x_1	x_3
x_2	x_4

a) Determine the matrix A such that the function $f(x) = Ax$ yields the image rotated by 90 degrees counterclockwise. That is, $f(x)$ should correspond to the image:

x_3	x_4
x_1	x_2

$e_1 \to e_2$

$e_2 \to e_4$

$e_3 \to e_1$

$e_4 \to e_3$

$$A = \begin{array}{c} \begin{matrix} e_2 & e_4 & e_1 & e_3 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

b) Determine the matrix A such that the function $f(x) = Ax$ yields the original image reflected left-right (as it would appear if viewed in a mirror).

x_3	x_1
x_4	x_2

$e_1 \to e_3$

$e_2 \to e_4$

$e_3 \to e_1$

$e_4 \to e_2$

$$A = \begin{array}{c} \begin{matrix} e_3 & e_4 & e_1 & e_2 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

2. Suppose the 4-vector c gives the coefficients of a cubic polynomial $p(t) = c_1 + c_2 t + c_3 t^2 + c_4 t^3$. Express the conditions

$$p(0) = 1$$
$$p(1) = 2$$
$$p'(0) = -p'(1)$$

as a set of linear equations of the form $Ac = b$. Give the sizes of A and b, as well as their entries.

$p(0) = 1 \Rightarrow c_1 = 1$

$p(1) = 2 \Rightarrow c_1 + c_2 + c_3 + c_4 = 2$

$p'(t) = c_2 + 2c_3 t + 3c_4 t$

$p'(0) = c_2$

$p'(1) = c_2 + 2c_3 + 3c_4$

$p'(0) = -p'(1) \Rightarrow c_2 = -c_2 - 2c_3 - 3c_4$

$\Rightarrow 2c_2 + 2c_3 + 3c_4 = 0$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 3 \end{bmatrix}}_{\substack{3 \times 4 \\ A}} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}}_{\substack{4 \times 1 \\ c}} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}_{\substack{1 \times 3 \\ b}}$$