Name: Solutions

1. Let a and b be vectors in $\mathbb{R}^n$. Carefully show that

$$(a+b)^T(a-b) = ||a||^2 - ||b||^2$$

$$(a+b)^T(a-b) = a^Ta + a^T(-b) + b^Ta - b^Tb$$

$$= ||a||^2 - a^Tb + a^Tb - ||b||^2$$

$$= ||a||^2 - ||b||^2$$

2. Let v and b be vectors in $\mathbb{R}^n$. Find the value of the number α that minimizes the norm squared

$$\|\alpha v - b\|^2.$$

(BTW: by solving this you are answering the following question: what multiple of v is closest to b?)

$$\|\alpha v - b\|^2 = (\alpha v - b)^T(\alpha v - b)$$

$$= \alpha^2 v^T v - 2\alpha b^T v + b^T b$$

$$= \alpha^2 \|v\|^2 - 2\alpha b^T v + \|b\|^2$$

$$\frac{d}{d\alpha}\|\alpha v - b\|^2 = 2\alpha\|v\|^2 - 2b^T v$$

Set derivative equal to 0: $2\alpha\|v\|^2 - 2b^T v = 0$

$$\Rightarrow \boxed{\alpha = \frac{b^T v}{\|v\|^2}}$$

3. Let x and y be boolean feature vectors (entries are 0 or 1) of symptoms exhibited by patients X and Y respectively. It turns out that $x^T y = 3$. What does this fact mean in everyday language?

The patients have 3 symptoms in common.