Name:

1. A function $f : \mathbb{R}^n \to \mathbb{R}$ is linear if it satisfies two properties related to vector addition and scalar multiplication. State the two properties.

$f(x+y) = f(x) + f(y)$ for all x, y in $\mathbb{R}^n$

$f(\alpha x) = \alpha f(x)$ for all numbers α and all x in $\mathbb{R}^n$.

2. Consider the function $f : \mathbb{R}^3 \to \mathbb{R}$ defined byt

$$f(x) = \begin{cases} 1 & \text{if each } x_i > 0,\ i = 1, 2, 3. \\ 0 & \text{otherwise.} \end{cases}$$

For example $f(1, 1, -1) = 0$ and $f(2, 1, 7) = 1$. Determine if f is linear function or not and fully justify your claim.

Let $x = (1,2,3)$ and $y = (1,1,5)$.

Then $x+y = (2,3,8)$.

Since $f(x) = 1$, $f(y) = 1$ we know $f(x) + f(y) = 1+1$
$= 2$

But $f(x+y) = 1$. Since $f(x+y) \neq f(x) + f(y)$,

$\boxed{f \text{ is } \underline{\text{not}} \text{ linear.}}$

3. The function $f : \mathbb{R}^3 \to \mathbb{R}$ is linear. It satisfies $f(1, 0, 0) = 5$, $f(0, 1, 0) = -2$ and $f(0, 0, 1) = 9$. Compute the value $f(3, 2, 1)$. Hint: $3(1, 0, 0) + 2(0, 1, 0) = (3, 2, 0)$.

$(3,2,1) = 3(1,0,0) + 2(0,1,0) + (0,0,1)$

So $f(3,2,1) = 3f(1,0,0) + 2f(0,1,0) + f(0,0,1)$

$= 3 \cdot 5 + 2 \cdot (-2) + 9$

$= \boxed{20}$