

Name:

1. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is linear if it satisfies two properties related to vector addition and scalar multiplication. State the two properties.

2. Consider the function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 & \text{if each } x_i > 0, i = 1, 2, 3. \\ 0 & \text{otherwise.} \end{cases}$$

For example $f(1, 1, -1) = 0$ and $f(2, 1, 7) = 1$. Determine if f is linear function or not and fully justify your claim.

3. The function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ is linear. It satisfies $f(1, 0, 0) = 5$, $f(0, 1, 0) = -2$ and $f(0, 0, 1) = 9$. Compute the value $f(3, 2, 1)$. Hint: $3(1, 0, 0) + 2(0, 1, 0) = (3, 2, 0)$.