

1. Compute the determinant of

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

↻ swap
3 swaps → I

$$\det(A) = (-1)^4 \det(I) = 1$$

2. Compute the determinant of

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 4 & 4 & 4 & 4 & 4 & 4 \end{bmatrix}$$

$$\det B = 1 \cdot 2 \cdot 1 \cdot 3 \cdot 1 \cdot 4 = 24$$

3. Using a touch of elimination, compute the determinant of

$$C = \begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

$$C \rightarrow \begin{bmatrix} 1 & 3 & 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & -5 \end{bmatrix} \quad \det(C) = (-5)^3 = -125$$

4. Compute the determinant of ABC where A , B and C are the matrices above.

$$\begin{aligned} \det(ABC) &= \det(A) \det(B) \det(C) = -24 \cdot 125 \\ &= -3000 \end{aligned}$$