Name: Solutions

1. Use the method of elimination to reduce the following matrix to echelon form.

$$A = \begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 4 & 6 & 7 & 5 & 14 \\ -2 & -3 & -5 & 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 4 & 6 & 7 & 5 & 14 \\ -2 & -3 & -5 & 2 & -3 \end{bmatrix} \xrightarrow[R_3 + R_1]{R_2 - 2R_1} \begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 0 & 0 & -1 & 3 & 2 \\ 0 & 0 & -1 & 3 & 3 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_2} \begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 0 & 0 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Determine all elements of the null space of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 4 & 6 & 7 & 5 & 14 \\ -2 & -3 & -5 & 2 & -3 \end{bmatrix}$$

from the previous problem.

Reduced form

$$\begin{array}{ccccc} p & f & p & f & p \end{array}$$
$$\begin{bmatrix} 2 & 3 & 4 & 1 & 6 \\ 0 & 0 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

solve $\begin{bmatrix} 2 & 4 & 6 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_5 = 0,\ x_3 = 0$

$2x_1 = -3 \Rightarrow x_1 = -3/2$

$$\boxed{n_1 = \begin{bmatrix} -3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

solve $\begin{bmatrix} 2 & 4 & 6 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix}$

$x_5 = 0$

$-x_3 + 2x_5 = -3 \Rightarrow x_3 = 3$

$2x_1 + 4 \cdot 3 = -1 \Rightarrow x_1 = \frac{-13}{2}.$

$$\boxed{n_2 = \begin{bmatrix} -13/2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}}$$

$\boxed{\text{Nullspace: all } c_1 n_1 + c_2 n_2 \text{ with } c_1, c_2 \in \mathbb{R}.}$

3. One solution of $Ax = (2, 4, -2)$ is given by $(1, 0, 0, 0, 0)$. What are all the other solutions?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -13/2 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

$c_1, c_2 \in \mathbb{R}.$