1. Let

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}.$$

Show that x_1, x_2 and x_3 are linearly dependent two different ways:

a) Find coefficients β_1, β_2, β_3 such that $\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = 0$.

b) Write x_1 as a linear combination of x_2 and x_3.

2. Let

$$y_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad y_3 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

a) Show that y_1, y_2 and y_3 are linearly independent. That is, show that if β_1, β_2 and β_3 are numbers such that $\beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 = 0$ then, in fact, $\beta_1 = \beta_2 = \beta_3 = 0$.

b) Briefly explain why y_1, y_2 and y_3 form a basis for $\mathbb{R}^3$. Your answer should be one sentence.

c) Because these vectors form a basis for $\mathbb{R}^3$, and because $z = (2, 1, 3)$ is a vector in $\mathbb{R}^3$, there is a unique linear combination $\beta_1 y_1 + \beta_2 y_2 + \beta_3 y_3 = z$. Find the numbers β_1, β_2 and β_3. This might be laborious and unpleasant. That's OK; I won't make you do stuff like this often.

3. Your book refers to something it calls the indepdendence-dimension inequality: "A linearly independent collection of n vectors has at most n elements".

a) Consider

$$a_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

Explain how you know, without doing any work, that this collection is linearly dependent.

b) Because the collection is linearly dependent, it has redundancy. Exhibit this redundancy by finding three different linear combinations of the vectors that give you 0. One of these will be super easy to find. One will take a little bit of work. Once you have that one, you can easily find infinitely many others, so locating a third will be a breeze!

c) Exhibit the redundancy differently by finding three different linear combinations of a_1, a_2 and a_3 that give you $(4, 7)$. **Hint:** Find one linear combination that works. Then use you answer from part **(a)** to help!

4. Suppose w_1, w_2 and w_3 are any vectors at all in $\mathbb{R}^{17}$. Let $v_1 = w_1 - w_2$, $v_2 = w_2 - w_3$ and $v_3 = w_3 - w_1$. Show that v_1, v_2 and v_3 are linearly dependent. **Hint:** find an explicit linear combination that yields zero.

5. Text: 5-4

6. Text: 5-5 modified as follows. Suppose a and b are any n-vectors. Find a formula in terms of a and b for a scalar γ such that $a - \gamma b$ is perpendicular to b. Then draw a picture of a, b, and $a - \gamma b$ when $a = (0, 1)$ and $b = (1, 1)$.