

1. Text 12.2 Modified. Suppose that the $m \times n$ matrix Q has orthonormal columns.
 - a) Why do you immediately know that $m \geq n$ and hence Q is tall or square?
 - b) Show that $\hat{x} = Q^T b$ is the vector that minimizes $J(x) = \|Qx - b\|^2$. Do this by setting up the normal equations directly.
2. Supplemental 12.6 Hint: Your matrix A will be 6×4 and will have a lot of 0's and 1's.
3. Suppose we want to compute the coefficients of the quadratic $p(t) = c_1 + c_2 t + c_3 t^2$ that is the least squares fit to the following (t, y) data points: $(0, 0)$, $(1, 8)$, $(3, 8)$, $(4, 20)$.
 1. Formulate the problem in the form of $Ac = b$. That is, find the matrix A and the vector b . You don't need to solve this system.
 2. Now formulate the normal equation $A^T A c = A^T b$ that you would use to find these best-fit coefficients. Again, do not solve the system. (But don't worry, there is a Julia problem that tackles this!)
4. Text: problem 12.4. Hint: you will find that the square roots of the weights are important. For part (b), remember that the columns of C are linearly independent if and only if the only solution of $Cx = 0$ is $x = 0$.