Now the big moment: matrix, vector multiplication

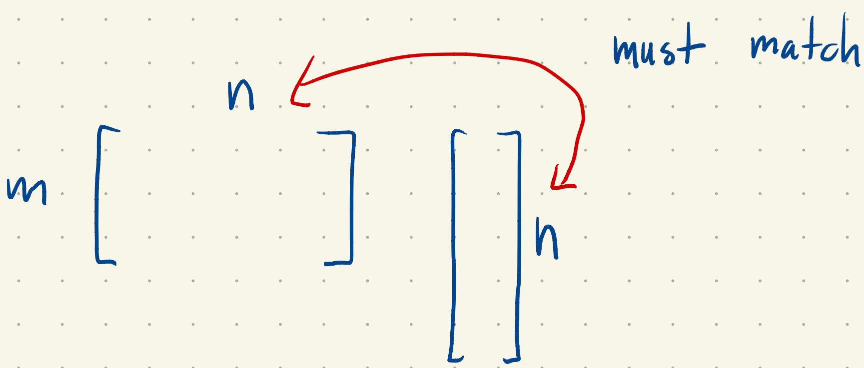
$$2a - 3b + 5c = 3$$
$$-a + b + 2c = 5$$

$$a\begin{bmatrix} 2 \\ -1 \end{bmatrix} + b\begin{bmatrix} -3 \\ 1 \end{bmatrix} + c\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$A x = b$$



A couple of perspectives

1) column perspective

$$\underbrace{\begin{bmatrix} a_1 & \cdots & a_k \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix}}_{x} = x_1 a_1 + \cdots + x_k a_k$$

"liner combination of colums of A, where coeficients come from x"

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

2) Row perspective

$$A\cancel{x} = \begin{bmatrix} b_1^T \\ \vdots \\ b_m^T \end{bmatrix} x = \begin{bmatrix} b_1^T x \\ \vdots \\ b_m^T x \end{bmatrix}$$

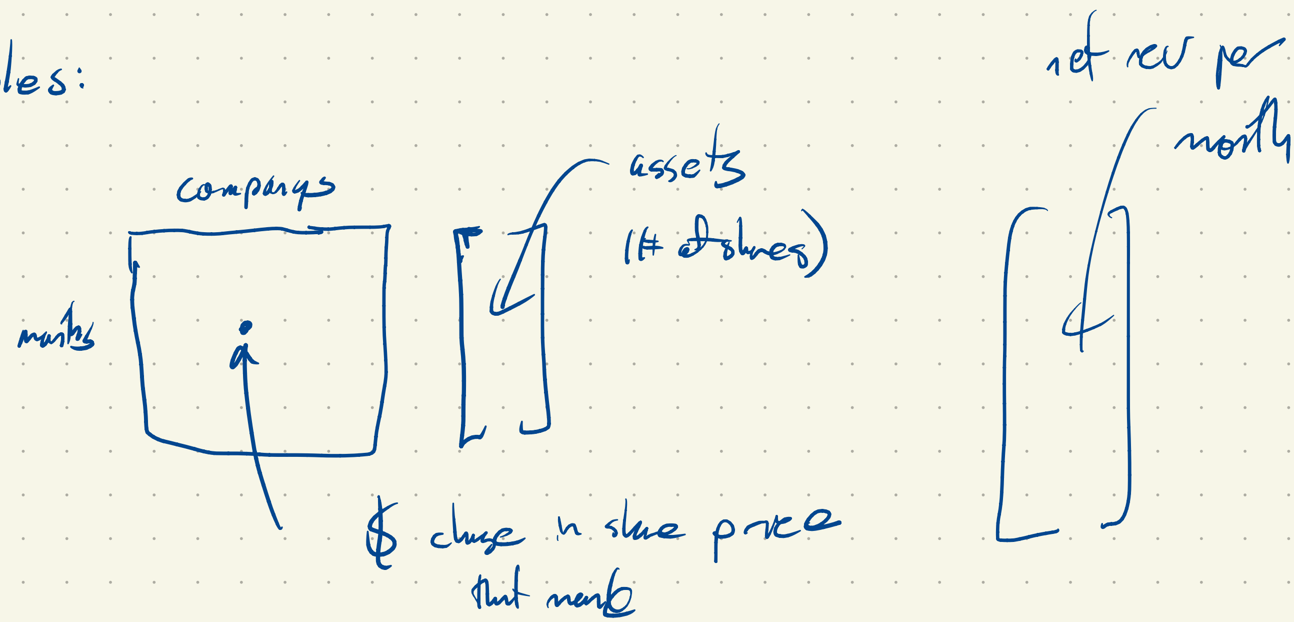
The j^{th} entry of Ax is row j dotted with x.

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 - 6 + 5 \\ -1 + 2 + 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$(Ax)_i = \sum_{j=1}^{n} A_{ij} x_j \qquad A \ m \times n, \quad x \in \mathbb{R}^n$$

Examples:

$A =$



$$\begin{bmatrix} a_1 & \cdots & a_k \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_k \end{bmatrix} = \underbrace{x_1 a_1 + \cdots x_k a_k}_{\text{mixed signal w/ weights}}$$

audio time signals

weights

$$\begin{array}{cc} & n+1 \\ n & \begin{bmatrix} -1 & 1 & & 0 \\ & -1 & 1 & \\ & & -1 \; 1 \; \ddots & \\ 0 & & & -1 \; 1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ \vdots \\ x_{n+1} \end{bmatrix} n+1 = \begin{bmatrix} x_2 - x_1 \\ \vdots \\ x_{n+1} - x_n \end{bmatrix}$$

"discrete derivative"

$$\begin{bmatrix} 1 & t_1 & t_1^2 & \cdots & t_1^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & t_k & t_k^2 & & t_k^n \end{bmatrix} \begin{bmatrix} c_0 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} c_0 + c_1 t_1 + c_2 t_1^2 + \cdots + c_n t_1^n \\ \vdots \\ \vdots \end{bmatrix}$$

Vandermonde, next lab!

Misc:

$$0x = 0$$

$$Ix = x$$

$$\begin{bmatrix} e_1 & \cdots & e_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 e_1 + \cdots + x_n e_n$$

$$A e_k = [a_1 \cdots a_n] e_k = 0a_1 + \cdots + 1a_k + \cdots 0a_n = a_k$$

(selects kth column)

Average of columns: $A \vec{1}$

$$(A+B)x = Ax + Bx$$

$$(cA)x = c(Ax) = A(cx)$$

almost as nice as can be!

$$A(x+y) = Ax + Ay$$

$$A(cx) = c(Ax)$$

$A(\alpha x + \beta y)$
$= \alpha Ax + \beta Ay$ (!)

Given a matrix A, $m \times n$,

we can associate with it a map $\mathbb{R}^n \to \mathbb{R}^m$

$$x \longmapsto Ax$$

Things you can represent:

all the above examples

assets $\longrightarrow$ revenue per month for markets

signal $\longrightarrow$ discrete derivative

coeffs $\longrightarrow$ polynomial evaluated at points

vector $\longrightarrow$ itself (I)

Heres some more

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}$$

permutation matrix

row $i \longrightarrow$ row j

if column i is e_j