Matrices:

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 3 & -1 & 2 & 6 \\ 4 & 1 & 9 & 3 \end{bmatrix} \qquad \underset{\text{3 rows}}{3} \times \underset{\text{columns}}{4}$$

$$A_{2,4} = 6$$

$$A_{3,2} = 9$$

$$A_{\underset{\text{row}}{i}\,\underset{\text{column}}{j}}$$

In general $\underbrace{m \times n}$

dimensions of the matrix

We identify vectors with matrices with one column

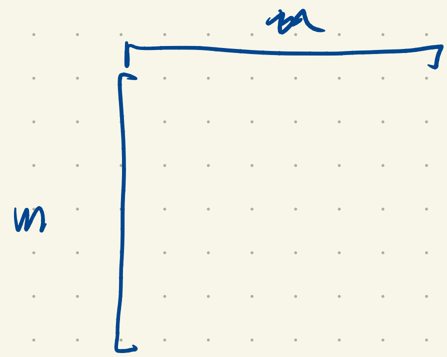
$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

by contrast, $\begin{bmatrix} 4 & 1 & 9 & 3 \end{bmatrix}$ is a <u>row</u> vector.

Sometimes vectors are called column vectors.

Sorta like tables in spreadsheets

examples: images:



$m \times n$ pixels with values A_{ij}

Lists of vectors.

$a_1, \dots, a_k$ vectors $\left[\, a_1 \mid a_2 \mid \cdots \mid a_k \,\right]$

e.g. $a_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $a_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $a_3 = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 4 & 9 \\ 1 & 1 & 3 \end{bmatrix}$$

Block notation $\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$

e.g. QR: $\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \rightarrow \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$

e.g. rows: Palmer — Jan — — . . Dec — avg monthly temp.

Juneau

Anchorage

Homer

e.g. Contingency tables

		dog ownership				
		0	1	2	3	4+
cat ownership	0	# of people				
	1					
	2					
	3					
	4+					

Block matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 7 & 9 \\ 6 & 8 & 10 \end{bmatrix}$$

$$C = \begin{bmatrix} 11 & 12 \end{bmatrix} \qquad D = \begin{bmatrix} 13 & 14 & 15 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 & 7 & 9 \\ 3 & 4 & 6 & 8 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

Frequently: $a_1, \dots, a_n$ vectors $\begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix}$

$b_1, \dots, b_n$ row vectors $\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$

Does $\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}$ make sense? $\begin{bmatrix} b_1 & \cdots & b_n \end{bmatrix}$?

A matrix is <u>square</u> if $m = n$

It is <u>tall</u> if $m > n$ and wide if $m < n$.

Super important square matrix:

$$I = \left.\underbrace{\begin{bmatrix} 1 & 0 & - & - & 0 \\ 0 & 1 & 0 & - & - \\ 0 & 0 & 1 & 0 & - \\ & & & \ddots & \\ 0 & - & - & 0 & 1 \end{bmatrix}}_{n}\right] n \text{, identity}$$

Sometimes I_n, given by context.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

diagonal.

It's called the identity because when we learn how to multiply, it will act like #1.

We denote the matrix of all 0's by 0.
Shape from context.

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

what is $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 \\ 4 & 5 & 6 & 0 & 1 \end{bmatrix}$$

A diagonal matrix:

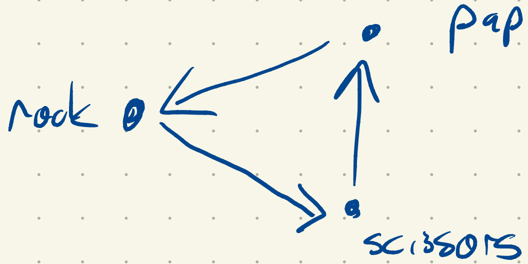
$$\operatorname{diag}(3,7,-1,6) = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

All entries are zero, except on the diagonals

$$I = \operatorname{diag}(1,1,1,1).$$

Linear Algebra. I

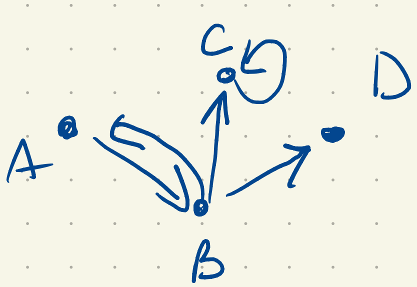
Directed Graphs



$$\begin{array}{cc} & \begin{array}{ccc} \text{rock} & \text{s} & \text{p} \end{array} \\ \begin{array}{c} \text{rock} \\ \text{scissors} \\ \text{paper} \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

rock's arrows

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$