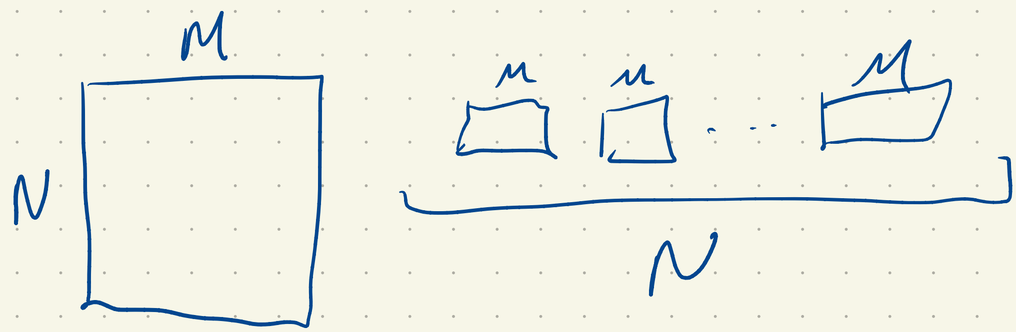
Last class: a vector is a list of numbers

examples: physics: displacement (position), velocity, acceleration, force — x, y, z components

ubiquitous: time series

finance: portfolio asset counts

also: images



each pixel $0 .. 1$

$0 .. 255$

RGB colors (r, g, b)

RGB image: $3MN$ numbers in order

AI (LLM)

latent space 768 numbers, a germ of an idea.

And lots more: see text (and I'll often do this)

Some vector operations:

(Note: no $\vec{a}$)

$a = (1, 3, 7, 5)$

indexing: $a_2 = 3$

$a_3 = 7$

$a_4 = 5$

Subsetting

$$a_{2:3} = (3, 7)$$

Concatenation

$$b = (2, -3, 9)$$

$$(a, b) = (1, 3, 7, 5, 2, -3, 9)$$

All zero vector: $0_n = \underbrace{(0, \ldots, 0)}_{n \text{ times}}$

All ones vector $\vec{1}_n = \underbrace{(1, 1, \ldots, 1)}_{n-\text{times}}$

0 or $\vec{1}$ if n is understood.

Standard basis vector $e_k = \underbrace{(0, \ldots, 0, \underset{\substack{\uparrow \\ \text{slot } k}}{1}, 0, \ldots, 0)}_{n \text{ understood}}$

Fundamental Vector Operations

1) Vector addition

2) scalar multiplication.

$$a = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$$

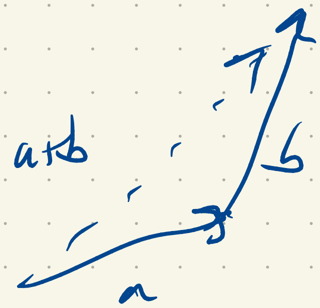
$$a + b = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$$ (just add entry-wise)

if portfolio assets, you just combined portfolios

if time series of pressures for sound waves,
you just superimposed the sound waves

if images, you just layed one image on top of another

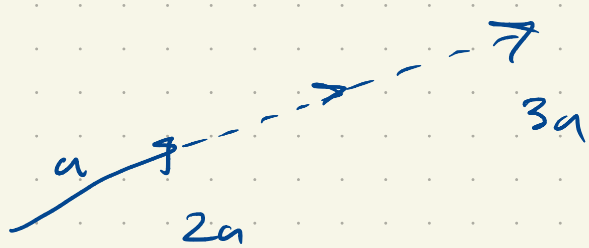
If displacements:



Scalar mult:

$$7 \cdot \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ 7 \\ 28 \end{bmatrix}$$

You just scale each entry.



audio signal:
louder, quieter

portfolios:
uniform inc/dec
that maintains
relative ratios

These two operations get along:

a, b, c vectors of same dimension, α a number
β

$$-a = (-a_1, \dots, -a_n)$$

$$a + b = b + a$$

$$(a+b)+c = a + (b+c)$$

$$a + 0 = 0 + a = a$$

$$a + -a = 0$$

$$\alpha(\beta a) = (\alpha\beta) a$$

$$\alpha(a+b) = \alpha a + \alpha b$$

$$1 a = a$$

$$(\alpha+\beta) a = \alpha a + \beta a$$

We combine these two operations as follows

$$\underbrace{\alpha a + \beta b} \qquad \alpha, \beta \in \mathbb{R}$$

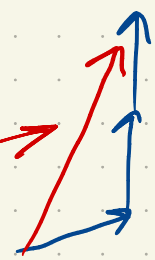
"a linear combination of a and b."

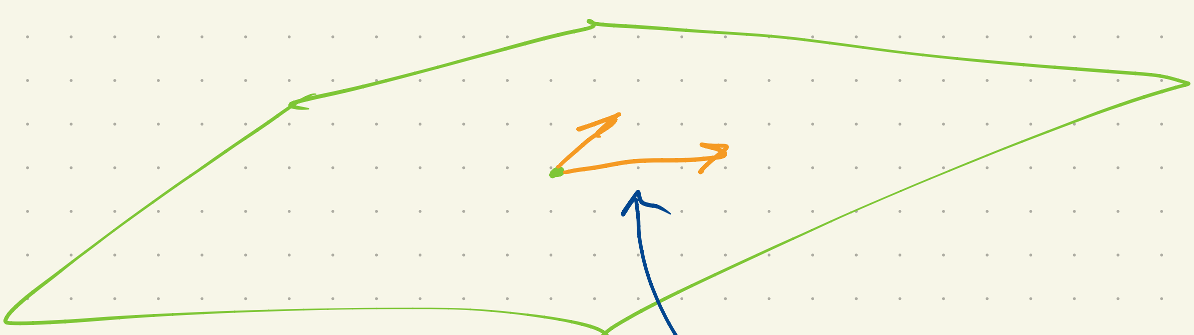
If $a_1, \dots, a_n$ are vectors $\alpha_1, \dots, \alpha_n \in \mathbb{R}$

$\alpha_1 a_1 + \dots + \alpha_n a_n$ is also a linear combo.

- Audio signals: combine at various volumes
- displacements

$0.5a + 2b$





If I give you these two vectors, what do you get by taking all linear combos?

Inner Product (duality)

$$a = (a_1, a_2, a_3, a_4)$$

$$b = (b_1, b_2, b_3, b_4)$$

$$a^T b = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$

$$a^T = [\ ,\ \dots ,\ ]$$

$$a = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$$(a^T)^T = a$$

$$\begin{bmatrix} & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

We'll see why T a bit later.

A <u>lot</u> of HW 1 is about seeing applications of this operation. What is it.

You can think of $a^T b$ as adding up the entries of b with weights coming from a,

e.g. $a = \vec{1}_4$ $\quad b = (b_1, b_2, b_3, b_4)$

$$a^T b = b_1 + b_2 + b_3 + b_4$$

e.g $a = e_3$

$a^T b = b_3$

e.g $a = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

$a^T b = \frac{b_1 + b_2 + b_3 + b_4}{4}$ (average)

e.g. a: portfolio assets

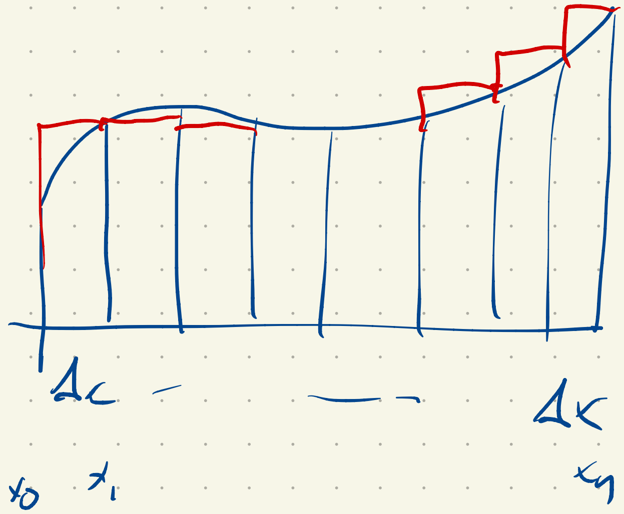
b: price per asset

$\left.\underbrace{a_1 b_1}_{\text{18 shares at \$46 a share AAPL}} + \cdots\right]$ total value of portfolio

18 shares at $46 a share

AAPL

e.g



$a = \Delta x \cdot \vec{1}$ $\quad$ $f(x_1)\Delta x + \cdots + f(x_n)\Delta x$ approx integral

$b_k = f(x_k)$

(total work, total energy production)

Some observations:

$$a^T b = b^T a$$

$$(\gamma a)^T b = \gamma(a^T b) \qquad a^T(\gamma b) = (\gamma b)^T a$$

$$= \gamma\, b^T a$$

$$= \gamma\, a^T b$$

$$(a+b)^T c = a^T c + b^T c$$

$$a^T(b+c) = a^T b + a^T c$$

$$a^T a = a_1^2 + \cdots + a_n^2$$ sum of squares

$x^2+y^2+z^2 \longrightarrow$ hmm. connection?

$a = (p_1, \dots, p_n)$

probabilities $0 \le p_i \le 1$, $p_1 + \dots + p_n = 1$

$b = (b_1, \dots, b_n)$

outcomes, b_k with probability p_k

e.g. a draws, b_k is the prize value.

(one b_k is 0, and p_k close to 1)

$a^T b$ is the expected winnings.