Name: Solutions

1. Suppose we want to find a polynomial $p(t) = c_1 + c_2 t$ passing through the three points with (t, y) coordinates given by $(-1, 2)$, $(0, 3)$ and $(2, 5)$. This can't be done, of course. Nevertheless, set up a system of the form $Ac = b$ to solve for the coefficients $c = (c_1, c_2)$. Your answer will consist of a 3×2 matrix A with numerical entries and a 3-vector b also with numerical entries.

oops! Yes it can...

$$\underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}}_{b}$$

2. Now set up the normal equation used to solve for the least squares solution. You do **not** need to solve the system. Your answer will be in the form $Bc = d$ where B is a matrix with numerical entries and d is a vector with numerical entries.

Normal equations:

$$A^T A x = A^T b$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \quad A^T A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

3. (Extra credit) Solve the system.

$$C = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} \quad C^{-1} = \frac{1}{14} \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}$$

$$C^{-1} \begin{bmatrix} 10 \\ 8 \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 42 \\ 14 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$