Name: Solutions

1. Suppose A is an invertible $n \times n$ matrix and that some generous person has provided a QR factorization, so Q is an orthogonal matrix, R is upper triangular matrix with no zeros on the diagonal, and $A = QR$. Given an n-vector b, state the **two** steps needed to solve $Ax = b$ for x using the QR factorization.

1) Let $w = Q^T b$

2) Solve $Rx = w$ using back substitution.

2. Find a right-inverse for the matrix

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}.$$

$C = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$$C^{-1} = \frac{1}{1\cdot 4 - 3\cdot 2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$$

Right inverse: $B = \begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \\ 0 & 0 \end{bmatrix}$

3. [Extra Credit] Use your right inverse from the previous problem to solve $Ax = b$ with $b = (2, 4)$

$$\begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 + 6 \\ 2 - 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Check: $A \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ ✓