Name:

1. Consider the vectors

$$v_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 1 \end{bmatrix}.$$

a) **(2 points)** Explain briefly how you know, without doing any work, that this is a linearly dependent collection of vectors.

b) **(4 points)** Explicitly show that the collection is linearly dependent by writing v_3 as a linear combination of v_1 and v_2.

c) **(2 points)** Find a set of numbers β_1, β_2 and β_3, not all zero, such that $\beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3 = 0$.

d) **(2 points)** One can compute that one solution of

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = \begin{bmatrix} 12 \\ 6 \end{bmatrix} \tag{1}$$

is $\alpha_1 = 2$, $\alpha_2 = 1$ and $\alpha_3 = 1$. Use your answer for part (c) to find a different linear combination of v_1, v_2 and v_3 that also equals $(12, 6)$. That is, find a different set of numbers α_1, α_2 and α_3 that also satisfies equation (1).