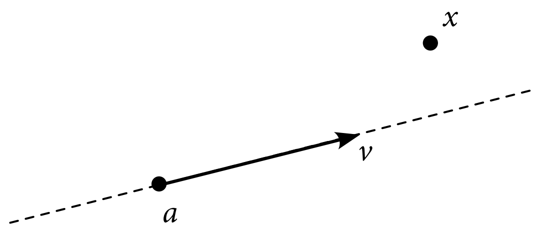
Name:

1. Let a be a vector and let v be a non-zero vector. As in the figure below, the line through a in the direction v is the set of all vectors of the form $a + sv$ where s is a number. Now consider some third vector x. We want to find the closest point z on the line to x.



a) The square of the distance from x to a point $a + sv$ on the line is given by

$$f(s) = ||a + sv - x||^2.$$

Carefully show that this square distance can **also** be written in the form

$$f(s) = ||a - x||^2 + 2s\, v^T(a - x) + s^2||v||^2$$

b) Recognizing this second expression for $f(s)$ as a quadratic in s, find the value of s that minimizes the square distance.

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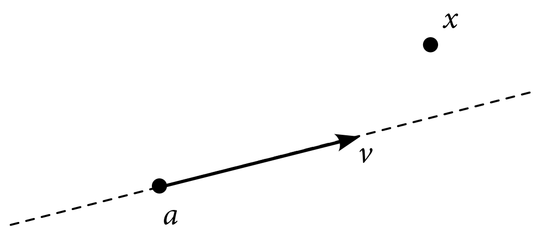

c) Using the value of s from part (b), one can show that the closest point z to x on the line is

$$z = a - \frac{1}{||v||^2}\left[(a-x)^T v\right] v.$$

You don't need to show this. However, use this expression for z to show that $z-x$ is perpendicular to v. Also, illustrate the perpendicularity by adding the point z in the figure above as well as the vector $z - x$.