Name: Solutions

1. (4 points) Consider the function $m : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$m(x) = \max(x_1, x_2, x_3).$$

Is m linear or not? If it is, find a vector c with $m(x) = c^T x$ for all x. If it is not, find a specific example (similar to what you did on your homework) where superposition fails.

m is not linear

$$m(1,0,0) = 1$$
$$m(0,1,0) = 1$$

$$(1,0,0) + (0,1,0) = (1,1,0)$$
$$m(1,1,0) = 1 \neq 2 = m(1,0,0) + m(0,1,0)$$

2. (4 points) Suppose f is linear and that we know:

$$f(1,2,2) = 5$$
$$f(2,1,1) = -3$$

Either compute $f(5,4,4)$ (with justification) or explain why this cannot be done with the information given.

Observe:

$$1\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 4 \end{bmatrix}$$

So. $f(5,4,4) = f(1,2,2) + 2f(2,1,1)$

$$= 5 + 2 \cdot (-3)$$

$$= -1$$