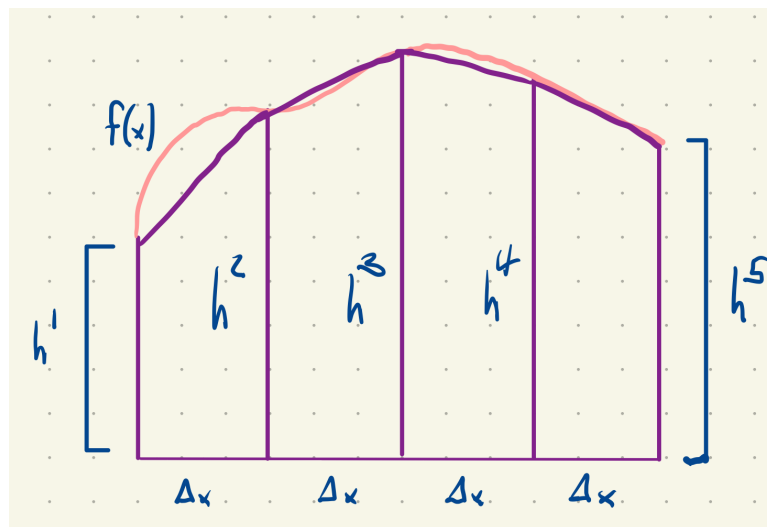


Name:

1. (4 points) The vector  $x$  has length 30 and each entry is either a 1 or a 0. Using an inner product, along with vector notation, write down a tidy little expression that equals the total number of ones in  $x$ .

$$\mathbf{1}_{30}^T x$$

2. (6 points) Recall the trapezoid rule for integrating functions, as in the figure below, where the integral is estimated using areas of trapezoids. The area of a trapezoid with width  $w$  and heights  $h_1$  and  $h_2$  is  $w(h_1 + h_2)/2$ .



Let  $h = (h_1, \dots, h_5)$  be the vector of heights in the figure, and assume that  $\Delta x = 1/4$ . Express the sum of the area of the trapezoids in the form  $c^T h$ . That is, find the vector  $c$ . A complete answer expresses  $c$  as a vector of 5 explicit numbers.

$$\begin{aligned} \text{Area: } & \frac{\Delta x}{2} (h_1 + h_2) + \frac{\Delta x}{2} (h_2 + h_3) + \frac{\Delta x}{2} (h_3 + h_4) + \frac{\Delta x}{2} (h_4 + h_5) \\ & = \frac{\Delta x}{2} [h_1 + 2h_2 + 2h_3 + 2h_4 + h_5] \\ & = \frac{1}{8} (1, 2, 2, 2, 1)^T h \end{aligned}$$

$$c = \left( \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8} \right)$$