F314 Linear Algebra: Quiz 10 April 26, 2024 Name: Solutions

1. Compute the determinant of

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

swap

swap $\to I$

swap

$$\det(A) = (-1)^3 = \boxed{-1}$$

2. Compute the determinant of

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}.$$

$$\det A = 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 = \boxed{8}$$

3. Using a touch of elimination, compute the determinant of

$$C = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}.$$

$R_2 = R_2 - 2R_1$

$R_4 = R_4 - 2R_3$

$R_6 = R_6 - 2R_5$

determinant does not change

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix} \to (-3)^3 = \boxed{-27}$$

4. Compute the determinant of ABC where A, B and C are the matrices above.

$$\det(ABC) = \det(A) \det B \det C = -1 \cdot 8 \cdot (-27) = \boxed{216}$$