$\det(A) \rightarrow$ number

$\uparrow$

square

$\det(A) \neq 0$

$\Leftrightarrow$ A is invertible

$$\det\begin{pmatrix} 1 & 0 \\ 0 & A' \end{pmatrix} = \det(A')$$

$$\det\begin{pmatrix} 1 & 0 \\ * & A' \end{pmatrix} = \det(A')$$

$$\det(AB) = \det(A)\det(B)$$

$Q \leftarrow$ orthogonal matrix

$Q^T Q = I$

$Q^{-1} = Q^T$

in fact $\det(Q) = \pm 1$

$$\det(A^T) = \det(A)$$

$$\underline{1} = \det(I) = \det(Q^T Q) = \det(Q^T)\det(Q)$$

$$= \det(Q)^2$$

$$\det(Q)^2 = 1 \Rightarrow \det(Q) = \pm 1$$

L is lower triangular $\quad \det(L) \leftarrow$ product of diagonal entries

U is upper triangular $\quad \det(U) \leftarrow$

$$\det(L^T) = \det(L)$$

$$\det(U^T) = \det(U)$$

$$A = LU \qquad \det(A) = \det(L)\det(U)$$

$$A^T = U^T L^T \qquad \det(A^T) = \det(U^T)\det(L^T)$$

$$= \det(U)\det(L)$$

X is an exchange matrix.

$$\det(X) = -1$$

$$X \cdot X = I$$

$$X^{-1} = X^T$$

$$X^{-1} = X$$

$$\det(X^T) = \det(X) = -1$$

$$PA = LU$$

$$X_1 X_2 \cdots X_k A = LU$$

$$A = X_k X_{k-1} \cdots X_2 X_1 LU$$

$$\det(A) = (-1)^k \det(L) \det(U)$$

$$A^T = U^T L^T X_1 X_2 \cdots X_k$$

$$\det(A^T) = \det(U^T) \det(L^T) (-1)^k$$

$$= (-1)^k \det(L) \det(U) = \det(A)$$

$$\det\begin{pmatrix} 1 & 0 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \det\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix}$$

$$A \to A^T \xrightarrow{\text{swap}} B^T \to B$$

$$\det\begin{pmatrix} 0 & 1 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = -\det\begin{pmatrix} 1 & 0 & 0 \\ 5 & 4 & 6 \\ 8 & 7 & 9 \end{pmatrix} = -\det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix}$$

$$\det\begin{pmatrix} 0 & 0 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = -1\det\begin{pmatrix} 0 & 1 & 0 \\ 4 & 6 & 5 \\ 7 & 9 & 8 \end{pmatrix} = +1\det\begin{pmatrix} 1 & 0 & 0 \\ 6 & 4 & 5 \\ 9 & 7 & 8 \end{pmatrix}$$

$$= +\det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{pmatrix}$$

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = a_{11}\det\begin{pmatrix} 1 & 0 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

minor M_{11}

$$+\ a_{12}\det\begin{pmatrix} 0 & 1 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

M_{12}

$$+\ a_{13}\det\begin{pmatrix} 0 & 0 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

M_{13}

$$a_{11}\det\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix} - a_{12}\det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix} + a_{13}\det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix}$$

$$\det(A) = a_{11}\det(M_{11}) - a_{12}\det(M_{12}) + a_{13}\det(M_{13})$$

$$\det\begin{pmatrix} 4 & 5 & 6 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{pmatrix} = -\det\begin{pmatrix} 0 & 1 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$= +\det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix}$$

$$\downarrow$$

$$+\det M_{22}$$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \qquad (-1)^{i+j}$$

$$\det\begin{pmatrix} 4 & 5 & 6 \\ a_{21} & a_{22} & a_{23} \\ 7 & 8 & 9 \end{pmatrix} = -a_{21}\det\begin{pmatrix} 5 & 6 \\ 8 & 9 \end{pmatrix} + a_{22}\det\begin{pmatrix} 4 & 6 \\ 7 & 9 \end{pmatrix}$$

$$- a_{23}\det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix}$$

$$\det\begin{pmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 0 & 3 \\ 0 & 1 & 0 & 9 \\ 3 & 1 & 1 & 2 \end{pmatrix} = 3\det\begin{pmatrix} 2 & 3 & 4 \\ 0 & 0 & 9 \\ 3 & 1 & 2 \end{pmatrix}$$

$$+3\det\begin{pmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$$

$$= 3(-9)\det\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$$

$$+3\cdot(+1)\det\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$$

$$= (-27+3)\det\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$$

$$= -24\cdot(-7)$$

$$= 24\cdot 7$$

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

two terms

$2\times 2: \quad 2\cdot 1$

$3\times 3: \quad 3\cdot 2\cdot 1$

$4\times 4: \quad 4\cdot 3\cdot 2\cdot 1$

$5\times 5: \quad 5!$

$10\times 10:$ 3.6 million terms

$$PA = LU$$

A $n \times n$

$O(n^3)$ operations to form LU decomp.

$$A = P^T L U$$

1000 ops for 10×10

$$\det(A) = \det(P^T)\det(L)\det(U)$$

$$= \det(P) \cdot 1 \cdot \det(U)$$

$$= \det(P)\det(U)$$

$$A = QR$$

$$\det(A) = \det(Q)\det(R)$$

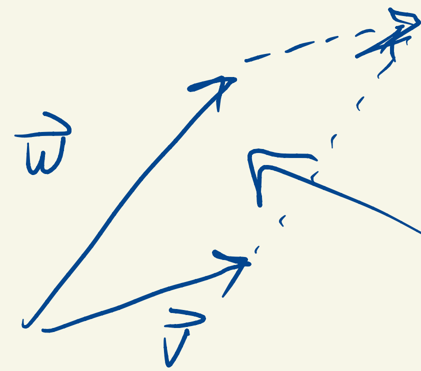
product of diagonal entries

$$\det(Q) = \pm 1$$

What does the determinant even mean?

It's the "volume" determined by the columns of A

$$A = \begin{pmatrix} \vec{v} & \vec{w} \end{pmatrix}$$



$\det(A) = \text{area}$ $\qquad \pm$

explain