$$-x_3 = 1$$

$$x_3 = -1$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \checkmark$$

there is a solution!

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$

is another solution!

$$\begin{bmatrix} 3 & 1 & 2 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$1 \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + (-3) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Is there a solution?

How many?

If no solution what's the best we can do.

How do you find solutions?

$$6\,CO_2 + 6\,H_2O \longrightarrow 1\,C_6H_{12}O_6 \;\; 6\,O_2$$

$$R = \begin{array}{c|cc} & CO_2 & H_2O \\ \hline H & 0 & 2 \\ C & 1 & 0 \\ O & 2 & 1 \end{array} \qquad P = \begin{array}{c|cc} & C_6H_{12}O_6 & O_2 \\ \hline H & 12 & 0 \\ C & 6 & 0 \\ O & 6 & 2 \end{array}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

x_1: amount of CO_2

x_2: amount of H_2O

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

y_1: amount of sugar

y_2: amount of O_2

$\underbrace{Rx} \rightarrow 3\times 1$

$Py \rightarrow 3\times 1$

$$Rx = P_y$$

"conservation of atoms"

amount of H

- - - - C

- - - - O

in $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$$R_x - P_y = 0$$

$$3\ \underbrace{\overbrace{\begin{bmatrix} R & -P \end{bmatrix}}^{4}}_{A}\ \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{z}\ 4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Az = 0$$

$$\downarrow$$

$$R_x - P_y = 0$$

$$\begin{bmatrix} R & -P \\ 1\ 0 & 0\ 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 6 \end{bmatrix}$$

CO_2

$$\begin{bmatrix} 0 & 2 & -12 & 0 \\ 1 & 0 & -6 & 0 \\ 2 & 1 & -6 & -2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}$$

$$1 \cdot x_1 = 6$$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{array}{c} H \\ C \\ O \end{array} \begin{bmatrix} 12 & 0 \\ 6 & 0 \\ 6 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 1 \\ 0 \end{bmatrix}$$

underdetermined $\quad m < n$

fewer equations than unknowns $\rightarrow$ non-uniqueness

overdetermined $\quad m > n$

more equations than unknowns $\rightarrow$ no solutions

A tall

wide

A

Matrix Matrix multiplication

first row

second col

first row second col

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}}_{B} = \begin{bmatrix} 14 & 0 \\ 4\cdot 1 + 5\cdot 2 + 6\cdot 3 & -3 \end{bmatrix}$$

$$\boxed{2} \times 3 \quad 3 \times \boxed{2} \rightarrow \boxed{2} \times \boxed{2}$$